PHY 142

COMPLETE

NOTE

Atomic & Nuclear Physics

- Theory of atomic structure
- Thompson, Rutherford and Bohr's theories
- The hydrogen atom
- properties of the electron
- Cathode Ray Oscilloscope (CRO) & millikan's experiment
- properties of the nucleus
- Natural Radioactivity
- Wave - particle duality of light
- X - rays and photoelectricity
- Thermionic emission
- Diode - valve

Atomic Structure

The concept of orbits disappear completely. The electrons can exist in any of a number of states characterized by energy, angular momentum and an orientation in space.

The electrons angular momentum is quantized as

$$L = l\hbar$$

L is a vector — l Angular momentum quantum number (Integer)

$\hbar = \frac{h}{2\pi}$ = reduced plank Constant

$l = 0, 1, 2, 3, \ldots$

The angular momentum vector may point in only $(2l+1)$ direction

n_r = radial quantum

$n = 0, 1, 2, 3$

NOTE: In Bohr's theory, Angular momentum is believed to be

$$L = l\hbar$$

Thus, Energy levels of hydrogen atoms are given by

$$E_n = \frac{-13.6\ eV}{n^2} \quad (n = 1, 2, 3)$$

$$= \frac{-13.6\ eV}{(n_r + l - 1)}$$

$$\boxed{n = n_r + l - 1}$$

n — principal quantum number

The generalized form of atomic struc

that is similar to the Bohr's theory but with the state defined by the different combination of the quantum numbers.

$n \geq l+1$; $l \leq n-1$

where $l = 0, 1, 2, 3$

l - small letter

$l = 2$

$= (-2, -1, 0, 1, 2)$

$= 5$ states

A state with a given "l" refers to a collection of $(2l+1)$ states all of which have the same angular momentum.

Thus, if $l=1$, we have 3 states $(-1, 0, 1)$

Measuring States

$n \geq l+1$

$l \leq n-1$ $\quad$ $l = 0, 1, 2, 3, \ldots n-1$

n	l (possible values)
1	0
2	0, 1
3	0, 1, 2

NOTE: (i) Atoms with the same n (no of states) have the same energy.

(ii) Atoms with the same 'l' have the same "angular momentum.

$$\underset{\text{Angular momentum}}{S = s\hbar} \pm \underset{\text{Spin quantum no.}}{\tfrac{1}{2}\hbar}$$

We can only have two spin quantum number namely $S = +\frac{1}{2}$ (up) ↑ or $S = -\frac{1}{2}$ (down) ↓

Magnetic Quantum Number:

m has values $= -l, 0, +l$

n	l		m	s	No of electron in l state	Total Particle
1	0	s	0	↑↓	2	$2e^{-s}$
2	0	s	0	↑↓	2	$8e^{-s}$
	1	p	-1, 0, -1	↑↓ ↑↓ ↑↓	6	
3	0	s	0	↑↓	2	$18e^{-s}$
	1	p	-1, 0, -1	↑↓ ↑↓ ↑↓	6	
	2	d	-2, -1, 0, 1, 2	↑↓ ↑↓ ↑↓ ↑↓	10	
4	0	s	0	↑↓	2	$32e^{-s}$
	1	p	-1, 0, 1	↑↓ ↑↓ ↑↓	6	
	2	d	-2, -1, 0, 1, 2	↑↓ ↑↓ ↑↓ ↑↓	10	
	3	f	-3, -2, -1, 0, 1, 2, 3	↑↓ ↑↓ ↑↓ ↑↓ ↑↓ ↑↓ ↑↓	14	

03-04-2014

HYDROGEN ATOM

- Hydrogen Atom
- Properties of the nucleus
- Natural radioactivity

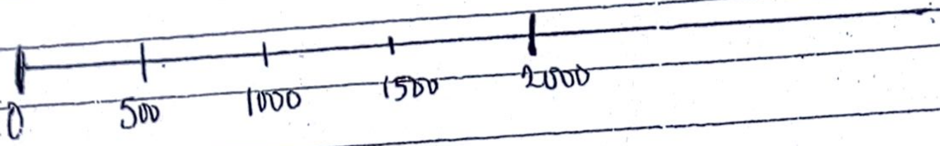
* When an atom reduces it energy level from $E_2 - E_1$ it releases energy 'hf'.

Thus $E_2 - E_1 = hf$

$$\Delta E = hf = \frac{hc}{\lambda}$$

Spectrometer is used to measure the properties of light over a specific portion of electromagnetic spectrum. It is used in spectroscopy to identify different materials. It is identified by its wave length.

If a spectrometer is pointed at a discharge tube fill with pure hydrogen, you will instead of a broad continuous spectrum, find a few lines of a particular colour.



visible region

G x UV IR

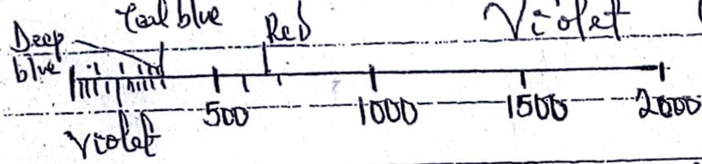
400 – 750nm

Red (λ = 656nm, H – alpha

Teal blue (λ = 486nm, H – beta

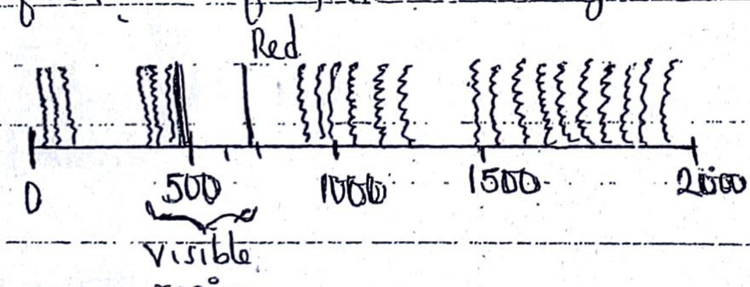
Deep blue (λ = 434nm, H – gamma

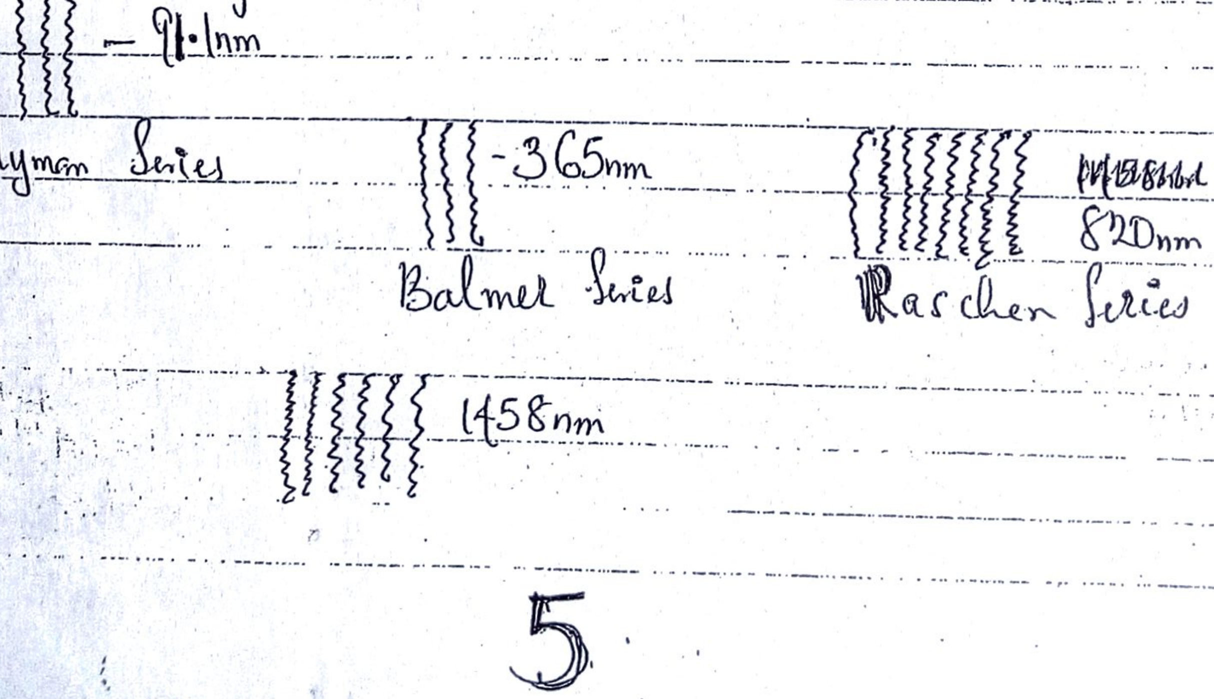
Violet (λ = 410nm H – delta



Other discrete lines i.e Seperate lines appear in the hydrogen spectrum but they lie outside the range of wavelength accesible by the normal human eye.

The figure below shows where the hydrogen lines are located as a function of their wavelength.





STIMULATED SPECTRA LINES OF HYDROGEN

In 1885, Balmer predicted the wave length in the Balmer group of the hydrogen spectrum.

$$\lambda = (364.56\ nm)\frac{n^2}{n^2-4} \quad \text{------} (1)$$

Where $n = 3, 4, \ldots$

Later, Rydberg generalized the Balmer formula in a way that also included all other line series in the hydrogen spectrum.

$$\frac{1}{\lambda} = R_H\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \quad \text{-----} (11)$$

Where $n_1 < n_2$ and are simple integers and R_H is Rydberg constant $= 1.097373 \times 10^7 m^{-1}$. $n_1 = 1$, $n_2 = 2, 3, 4, \ldots \infty$.

Therefore, smallest λ in each series must correspond to the largest value of n_2.

NOTE: Hydrogen gas can emit light when excited.

* For Lyman Series, the lowest (Series limit)

$n_1 = 1$, $n_2 = 2, 3, 4, \ldots$

Balmer Series

$n_1 = 2$, $n_2 = 3, 4, \ldots$

Paschen Series

$n_1 = 3$, $n_2 = 4, 5, \ldots$

Brackett Series

$n_1 = 4$, $n_2 = 5, 6, \ldots$

NOTE: Hydrogen gas can emit light when excited but the light cannot just have wavelength but it will only appear at well defined wavelengths described as RYDBERG FORMULA

Characteristic line spectra often serves as atomic "finger print" to detect the presence of specific chemical element.

Thus for any series n_2 and the λ vary inversely i.e

$n_2 \propto 1/\lambda$

The Smallest n_2 has the highest wavelength of the Series.

BOHR'S MODEL OF THE ATOM

Atom consists of nuclei which consist of positively charged proton and uncharged neutron, surrounded by negatively charge electron orbiting the nucleus and bound to it by Columb's Interaction.

Ionization of an atom is the removal of one or more electron resulting in a positively charged ion.

The typical size of an atom is of the diameter 10^{-10}m and the physical size of the atomic nucleus is 10,000 time smaller than the atom.

Hydrogen is the most abundant element in the universe and the simplest atom with one proton in the central nucleus and one electron orbiting it.

The motion in a circular orbit requires a constant centripetal acceleration. In this case, the CCA is provided by the Columb force.

$$F = \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$$

OR

$$\frac{e}{r^2} = \frac{\mu v^2}{r} \qquad \text{------- (iii)}$$

$$\mu = \frac{mM}{m+M} \qquad \text{------- (iv)}$$

$m = m_e = 9.109 \times 10^{-31}$ kg

$M = M_p = 1.6726 \times 10^{-31}$ kg

$\mu = 9.104 \times 10^{-31}$ kg

A electron moving in a circular orbit and experiencing constant CCA will quickly lose energy and move into the nucleus. i.e. radiate it, thus destroying the atom. That means the

Classical electromagnetic theory has failed in explaining the line spectra. Therefore, Neil Bohr region explain the line spectra by saying that "energy is radiated or absorbed not in a continuous manner but in the form of discrete packets or quanta having the magnitude, hf", where h is the planck's constant and f is the frequency of radiation.

An electron rotating in stationary orbit for which the E is E_2 can make a quantum jump i.e specific energy release.

$$\Delta E = hf = \frac{hc}{\lambda}$$

Bohr's model assumes that the angular momentum of the rotating electron must be an integral multiple of $\frac{h}{2\pi}$ i.e.

$$mvr = \frac{nh}{2\pi} \quad \text{---- (v)}$$

where $n = 1, 2, 3, \ldots$

Rem $P = h/\lambda$ uncertainity principle

$$P\lambda = h$$

where n is the principal quantum number.

The electrostatic attraction between the electron and the nucleus provides the necessarily centripetal force. i.e.

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2}{r^2}$$

$$v^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2}{mr} \quad \text{------- (vi)}$$

Sub. for v from (v)

$$mvr = \frac{nh}{2\pi}$$

$$v = \frac{nh}{2\pi mr} \qquad v^2 = \frac{n^2h^2}{(2\pi)^2 m^2r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2}{r^2}$$

$$\frac{n^2h^2}{(2\pi)^2 m^2r^2} = \frac{1}{4\pi\varepsilon_0} \times \frac{Ze^2}{mr}$$

$$r = \frac{\varepsilon_0 n^2h^2}{\pi m Ze^2} \quad \text{---- (vii)}$$

8

For Hydrogen, Z=1 and the smallest value of r will be when n=1 and this is known as Bohr's radius.

Thus, when n=1

$$r_1 = \frac{\varepsilon_0 h^2}{\pi m e^2}$$

$$= 0.529 \times 10^{-10}\ m$$

where $m = m_e$

K.E of an electron rotating in its orbit is

$$E_K = \frac{1}{2} mv^2$$

$$= \frac{1}{2} m \left(\frac{n^2 h^2}{4\pi^2 Z e^2} \right) \quad . \text{OR}$$

$$= \frac{1}{2} m \left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2}{mr} \right)$$

$$= \frac{1}{2} \left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2}{r} \right)$$

$$= \boxed{\frac{Ze^2}{8\pi\varepsilon_0 r}} \quad \text{------} \ (viii)$$

P.E of the electron also

$$P.E = \int \text{centripetal force} \times dr$$

$$E_p = \int \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2}{r^2}\, dr$$

Since $\frac{1}{4\pi\varepsilon_0}$ and Ze^2 are constant.

$$E_p = \int \frac{1}{r^2}\, dr = \int r^{-2}\, dr = \frac{-1}{r}$$

Thus, $E_p = \dfrac{-Ze^2}{4\pi\varepsilon_0 r}$ ---------- (ix)

Therefore, Total Energy E_n,

$$E_n = E_K + E_p$$

$$= \frac{Ze^2}{8\pi\varepsilon_0 r} - \frac{Ze^2}{4\pi\varepsilon_0 r}$$

$$E_n = \frac{-Ze^2}{8\pi\varepsilon_0 r} \quad - - - - - - - - - - (x)$$

The -ve sign shows that the energy is Decreasing.
Sub. for r from (xii) into (x)

$$r = \frac{\varepsilon_0 n^2 h^2}{\pi m Z e^2}$$

$$E_n = \frac{-Ze^2(\pi m Z e^2)}{8\pi\varepsilon_0(\varepsilon_0 n^2 h^2)} = \frac{-Ze^2(mZe^2)}{8\varepsilon_0^2 n^2 h^2}$$

$$E_n = \frac{-mZ^2e^4}{8\varepsilon_0^2 n^2 h^2} \quad - - - - - - - - (xi)$$

E_n is lowest for $n=1$ and E_n tends to zero as n tends to ∞ i.e

$$E_0 \rightarrow 0 \quad as \quad n \rightarrow \infty$$

EXAMPLE: For quantum state $n=n_2$, for an $\bar{e}$, jumping to another state $n=n_1$.

Solution

$$E_2 = \frac{-mZ^2e^4}{8\varepsilon_0^2 n_2^2 h^2} \qquad E_1 = \frac{-mZ^2e^4}{8\varepsilon_0^2 n_1^2 h^2}$$

There will bee emission in form of radiation

$$hf = E_2 - E_1$$

$$hf = \frac{-mZ^2e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

10-04-2014

Recall that

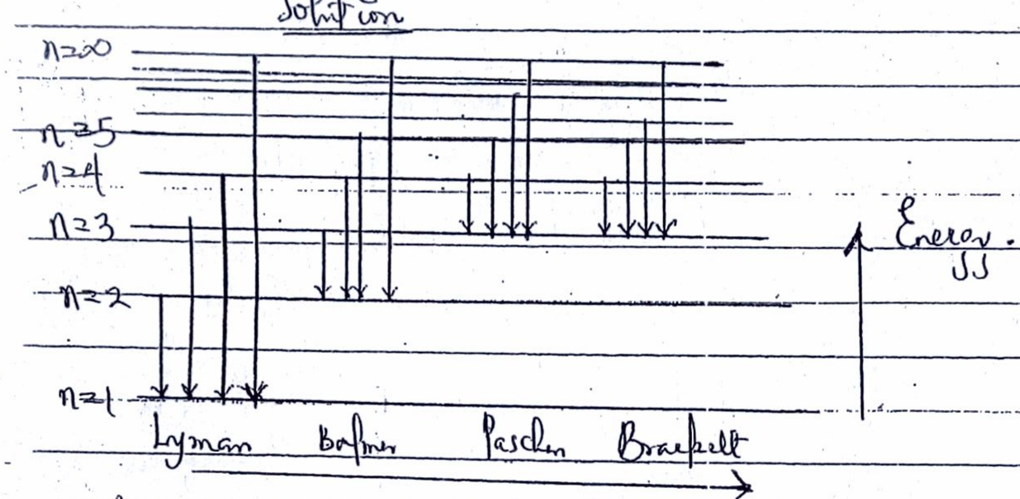
$$c = f\lambda \qquad f = c/\lambda$$

$$\frac{hc}{\lambda} = \frac{mZ^2e^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda} = \frac{mZ^2e^4}{8\varepsilon_0^2 c h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \Rightarrow \text{Rydberg Formula.}$$

EXERCISE: Calculate the shortest wavelength in the Layman series of the hydrogen atom.

Solution



$\frac{1}{\lambda}_{NJ} = 1.214 \times 10^{7}\ m$

Using $\frac{1}{\lambda} = R_H \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$

where $n_1 = 1$ and $n_2 = 2$

for Lyman Series

EXAMPLE: With what minimum energy should an atom be bombarded by an external electron so that on the excitation, the first Balmer line is emitted.

Solution

Using $hf = \frac{mZ^2e^4}{8\varepsilon_0^2h^2} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$

where $\varepsilon_0 = ...$ $Z = 1$

$n_1 = 2$ and $n_2 = 3$

$hf = 13.69\ eV$

∴ For an hydrogen atom, when $n=1$, $hf = -13.6\ eV$. Thus, E_n is lowest for $n=1$ and it is called the ground state of the atom and the higher levels are called excited states.

As $n \to \infty$

$E_\infty \to 0$

Thus, when $n = \infty$, $E_\infty = 0$ and the electron is longer bound to the nucleus.

The energy needed to remove an electron from an atom in its ground state is called its Ionization Energy. i.e. (The minimum

energy required to set an electron free)
Thus, $I.E = E_0 - E_1$.

10-04-2014.

NATURAL RADIOACTIVITY.

It was first observed by Bacquerel (1896) that uranium emitted radiation, capable of going through layers of matter, completely opaque to ordinary light, which could be detected by their property of ionizing gases.

Radon and Thorium are capable of emitting these radiations. These radiations are classified into three well defined groups namely.

(i) α-rays (ii) β-rays (iii) γ-rays

α-RAYS

α-rays are strongly ionizing i.e they can cause an electron to move to higher levels and also they are weak in penetration. They can be cut off (i.e. absorbed) by a thick sheet of paper and can be deflected by electrostatic and magnetic field as +vely charged particles. They are identified as a Helium nuclei i.e ^4_2H.

β-RAYS

β-rays are more penetrating but less penetrating but less ionizing than α-rays. They are also deflected by strong electrostatic and magnetic field as -vely charged particles. They are identified as high speed electrons i.e $_{-1}e$

γ-RAYS

They are highly penetrating but very weakly ionizing but they are undeflected by electrostatic and magnetic field showing them to be uncharged.

They consist of very high frequency E.M waves. Thus they have almost the same nature with X-RAYS.

The radiation effect changes in the nucleus due to emission according to Soddy - Fajan's displacement law.

According to this law, the emission of an α-particle reduces atomic mass of the nucleus by 4 units and the atomic number by 2 units.

i.e. $^{A}_{Z}X - ^{4}_{2}He \rightarrow ^{A-4}_{Z-2}Y$

$$^{A}_{Z}X \rightarrow ^{A-4}_{Z-2}Y + ^{4}_{2}He.$$

Also, due to the emission of a β-particle, the atomic number is increased by 1 unit while the atomic mass remains unchanged.

$^{A}_{Z}X - ^{0}_{-1}e \rightarrow ^{A}_{Z+1}Y$

$$^{A}_{Z}X \rightarrow ^{A}_{Z+1}Y + ^{0}_{-1}e$$

A series of α and β-particles are emitted until a stable product is attained which is an ISOTOPE.

The chain of these changes is called a radioactive series and the phenomenon is known as radioactivity.

(1) Radioactive elements are continuously and spontaneously broken down to new element with the emission of α-particles and β-particles accompanied by the emission of γ-rays in most cases.

(2) The no. of atoms that disintegrate in unit time is directly proportional to the number of unchanged radioactive atoms remaining.

If n is the no. of atoms of the radioactive element which has not disintegrated at time t, and if

$N_0 \xrightarrow{t} N$

dN of the atoms disintegrate in subsequent time dt, then $\frac{dN}{dt} \propto N$.

Thus, $\frac{dN}{dt} = -\lambda N$ ------ (1)

where λ = Decay / Disintegration constant.

$$dN/N = -\lambda\, dt \quad \text{------} \quad (II)$$

Recall that, $\left(\int dx/x = \log_e x\right)$

Thus,

integrating both sides

$$\int dN/N = -\lambda \int dt$$

taking natural log.

$$\log_e N = -\lambda t + A \quad \text{------} \quad (III)$$

If N_0 is the number of atoms initially present, i.e $t=0$

Thus, in (III)

$$\log_e N_0 = A \quad \text{----} \quad (IV)$$

putting (IV) in (III)

$$\log_e N = -\lambda t + \log_e N_0$$

$$\log_e N - \log_e N_0 = -\lambda t$$

$$\log_e \left(N/N_0\right) = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$N = N_0\, e^{-\lambda t} \quad \text{----} \quad (V)$$

EXAMPLE: If $N_0 = 20$ atoms

$t = 2\,\text{mins} = 2 \times 60\,\text{s}$

$N = ??$ $\quad \lambda =$

Solution

If $N = N_0/2$ i.e $t =$ half life

Sub. in (V)

$$\frac{N_0}{2} = N_0\, e^{-\lambda t}$$

$$1/2 = e^{-\lambda t_{1/2}}$$

$$0.5 = e^{-\lambda t_{1/2}}$$

taking natural log of both sides

$$\log_e 0.5 = \log_e e^{-\lambda t_{1/2}}$$

$$-0.693 = -\lambda t_{1/2}$$

Since $\log_e e = 1$

$$-0.693 = -\lambda t_{1/2}$$

$$t_{1/2} = \frac{0.693}{\lambda}$$

HALF - LIFE

This is defined as the time in which the number of atoms of the radioactive substance reduce to half its initial value.

Example:- The half life period of a nuclei is about 140 days. During this period, the average number of alpha emission 1 day for an initial mass of 1μg is about 12×10^{12} atoms. Assuming one emission per atom and density of the nuclei is $10^4\ kg\ m^{-3}$. Estimate the number of atoms in $1cm^3$ of the nuclei.

Solution

No of atoms that disintegrate for 140 days =

$2 \times 10^{12} \times 140$

Recall that.

$$\frac{N_o}{2} = 12 \times 10^{12} \times 140$$

$$N_o = 2 \times 12 \times 10^{12} \times 140$$

$$N_o = 336 \times 10^{13} \text{ atoms.}$$

Initially;

Initial mass $= 1\mu g = 10^{-9}\ kg$

Thus,

$10^{-9}\ kg$ contains 336×10^{13} atoms

Since $m/v = d$

$m =$ density $\times$ Volume

Converting $1cm^3$ to m^3

$= 10^{-6}\ m^3$

Mass $= 10^{-6}\ m^3 \times 10^4\ kg\ m^{-3}$

$= 10^{-6+4}\ kg = 10^{-2}\ kg$

Thus,

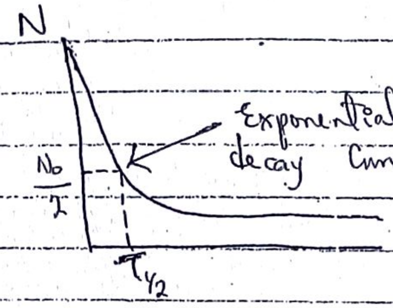
10^{-9} kg contains 336×10^{13} atoms

10^{-2} kg will contain x atoms

$$x = \frac{10^{-2} \times 336 \times 10^{13} \times 10^{9}}{10^{-9} \times 10^{9}}$$

$$= 336 \times 10^{-2+13+9}$$

$$= 3.36 \times 10^{22} \text{ atoms}$$



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PROPERTIES OF ELECTRONS

Electrons were discovered by J J Thompson in the 19th century. Some properties of electrons are:

(i) They have very low mass ($\frac{1}{1840}$ times the mass of a hydrogen atom.

(ii) They carry a tiny quantity of negative charge.

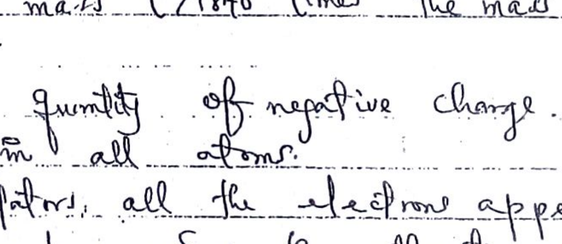
(iii) They are present in all atoms.

Furthermore, in insulators, all the electrons appears to be firmly bonded to the nucleus under the attraction of the unlike charges.

In metals, some of the electrons appear to be relatively 'free'.

CATHODE RAY OSCILLOSCOPE (C.R.O)

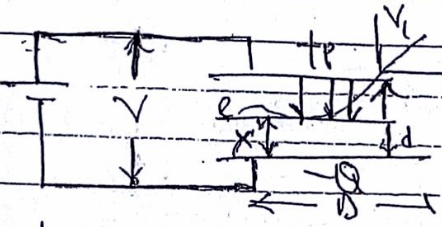
Before electrons were known to be carrying small charge, beams of electrons were called cathode rays because they came from inside the cathode.



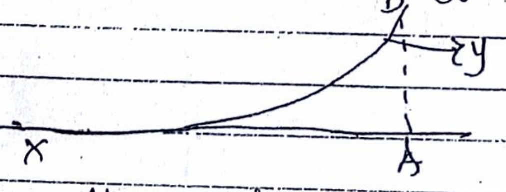
16

fast moving electrons are emitted from a heated cathode and these produces a sharp shadow of a maltese cross.
From this, it was discovered that electron travel on straight line and they produce heat when incident on a metal.
However, when a magnet is brought close to it, a deflection is observed.
NOTE: The direction of deflection is dependent on the pole of magnet.

ELECTRON MOTION IN ELECTRICAL & MAGNETIC FIELD.



Suppose a horizontal beam of electron moving with a velocity V, passes between two parallel plates P and Q. If the PD between the plates is V, and their distance apart is d, the field strength, $E = V/d$.
Therefore the force on an electron, e^- moving between the plates $F = eE = \frac{eV}{d}$ and is directed towards the +ve plate.
Since the field strength E, is vertical, no horizontal force acts on the electron entering the plates. The horizontal velocity V of the plate is not affected.



$$y = \tfrac{1}{2} at^2$$

where $a = \frac{F}{m} = \frac{eE}{m_e}$

putting $a = \frac{F}{m}$ in $y = \frac{1}{2}\left(\frac{eE}{m_e}\right)t^2$ ------- (1)

Horizontal displacement

$X_A = vt$ ---- (i)

Between the plates, e and me are constant. Combining (i) and (ii)

$\frac{1}{2} \cdot \frac{e}{m_e} \cdot E\left(\frac{x}{v}\right)^2$ ---- (iii)

$y = \frac{1}{2}\left[\frac{e}{m_e} \cdot \frac{E}{v^2}\right]x^2$

Since $E/m_e v$ is constant,

$y = \frac{1}{2}kx^2$

This is the parabolic equation. So, the path taken by the beam of electron is a parabolic path.

NOTE:

(i) When an electron moves into a uniform vertical electric field, it describes a parabolic path.

(ii) The horizontal motion of the electron is not affected by the field.

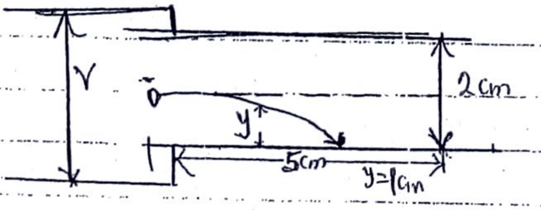
(iii) A charge gains energy when it moves in the direction of an electric field.

EXAMPLE: A beam of electrons moving with a velocity $1.0 \times 10^7\ ms^{-1}$ enters mid-way between two horizontal parallel plates PQ in a direction parallel to the plates. P & Q are 5cm long and 2cm apart and have a potential difference of V applied between them.

Calculate V if deflected downwards so that it just grazes the edge horizontally of a lower plate Q.

(Assume $e/m = 1.8 \times 10^{11}\ Ckg^{-1}$)

Solution.



$E = \frac{V}{d} = \frac{V}{2 \times 10^{-2}m}$

Recall that

$a = \frac{f}{m} = \frac{eE}{m_e}$

$= 1.8 \times 10^{11} \times \frac{V}{2 \times 10^{-2}m}$

$= 0.9 \times 10^{13}\ v$

Also, $a = 9 \times 10^{12}\ v$

But, $y = 1\ cm = 1 \times 10^{-2} m$ and

$x = vt$

$t = x/v$

$= \frac{5 \times 10^{-2}}{1 \times 10^{7}} = 5 \times 10^{-9}$

$y = 1 \times 10^{-2} m$

$x = 5 \times 10^{-2} m$

$t = 5 \times 10^{-9} s$

$y = \frac{1}{2} \frac{eE}{M_e v^2} x^2$

$V = \frac{2\, y\, m_e v^2 d}{e x^2}$

$\frac{M_e}{e} = 5.56 \times 10^{-12}$

$V = \frac{2 \times 1 \times 10^{-2} \times 5.56 \times 10^{-12} \times 1 \times 10^{14} \times 2 \times 10^{-2}}{25 \times 10^{-4}}$

$= 88.88\ V$

$\simeq 89\ V$

24-04-2014

WAVE - PARTICLE DUALITY NATURE OF MATTER

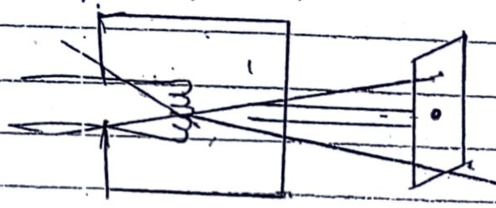
Matter sometimes behave as a wave and at other time as a particle. There are therefore two theories of matter.

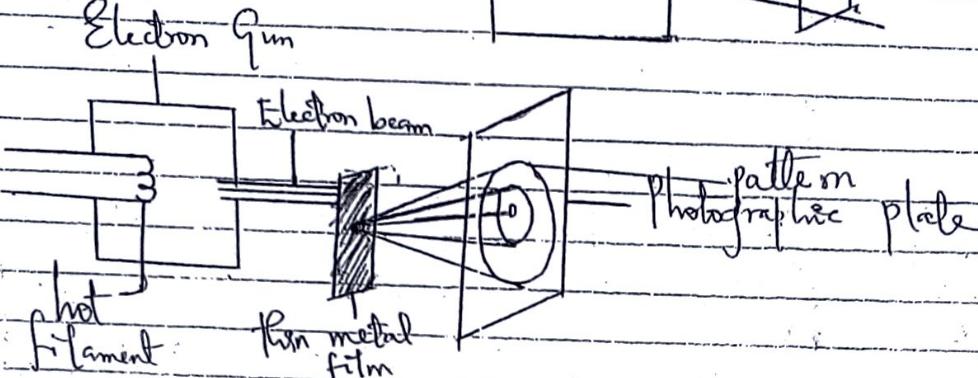
(i) the wave theory

(ii) the particle theory.

The wave nature of x-ray was established by x-ray diffraction experiment in the same way. Davisson and Germer experiment established the wave nature of electron. In both experiment, a beam of electrons emitted from a heated filament was made to fall on a layer of a thin metal film.

The electrons were Diffracted and the diffraction rings were produced on a photographic plate place behind the thin metal film.





ELECTRON DIFFRACTION EXPERIMENT

If the voltage V supplied to the filament at the anode was increased the velocity v of the electron increases.
Recall that,

$$E = eV \equiv \frac{1}{2}mv^2$$

$$eV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2eV}{m}}$$

Also, $\lambda = \frac{h}{mv}$

Sub. (v),

$$\lambda = \frac{h}{m\sqrt{\frac{2eV}{m}}}$$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

Since $eV = E$

Going back to the previous eqn,

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{h}{\sqrt{2mE}}$$

Thus from the relation above, $\lambda \propto 1/\sqrt{V}$ and when V is increased, the rings were then seen to become narrower. Hence the wavelength of the electron decreases with increasing electron voltage.

Later experiments show that protons, neutrons and other particles also have the wave property of diffraction.

PARTICLE NATURE OF MATTER

ASSIGNMENT

(i) Write briefly the phenomena that supports the wave nature of matter (ii) Particle nature of matter.

* Some experiments like electron diffraction etc indicates that matter behaves like a wave. But other experiments such as photoelectric effect and Compton effect experiments indicate that matter behaves like a stream of particles or photons.

These two theories seem to be incompatible but both have been shown to have validity.

This, matter appear to have a dual-nature. This is referred to as the wave-particle duality of the wave-particle paradox. In essence, it refers to the idea that light and matter have both wave and particle properties i.e light behaves either as a wave or a particle but not as both simultaneously.

EXAMPLE: Calculate the λ associated with the following objects: (i) electron moving with a velocity of 10m/s.

(ii) bullet of mass 0.01kg with a velocity of 400m/s.

(iii) An athlete of mass 50kg with velocity 10m/s.

Solution

(i) $M_e = 9.1 \times 10^{-31}$ kg , $V = 10^6$ m/s

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ J\cdot s}{9.1 \times 10^{-31} \times 10^6}$$

$$= 7.29 \times 10^{-10}\ m$$

(ii) $M = 0.01$ kg

$V = 400$

$$\lambda = \frac{\cancel{0.63 \times 10^{-34}}\ Js}{\cancel{1.658 \times 10^{-34}}}$$

$$\lambda = \frac{0.63 \times 10^{-34}\ J\cdot s}{0.1 \times 10^{-2} \times 400}$$

$= 1.658 \times 10^{-34} m$

(iii) $\lambda = \dfrac{6.63 \times 10^{-34} J\cdot s}{60 \times 10}$

$= 1.105 \times 10^{-36}$

EXAMPLE:

Electrons are accelerated with a potential difference of (i) 100V (ii) 400V (iii) 3600V.
Calculate the λ associated with the electron in each case.

Solution

(i) $e = 1.602 \times 10^{-19} C$

$M_e = 9.1 \times 10^{-31}$ kg

$h = 6.63 \times 10^{-34}$ J·s

$\lambda = \dfrac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-14} \times 100}}$

$= 1.21 \times 10^{-30}$ m

(ii) When $V = 400$,

$\lambda = \dfrac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-14} \times 400}}$

$= 6 \times 10^{-11} m$

(iii) when $V = 3600$

$\lambda = \dfrac{6.63 \times 10^{-34}}{3.25 \times 10^{-23}}$

$= 2.04 \times 10^{-11} m$

EXAMPLE:

An X-ray photon has a wavelength of 3.3×10^{-10} m. Calculate the (i) momentum (ii) Mass (iii) Energy of the particle associated with the photon which moves with a velocity c.

Solution

$V = c = 3 \times 10^{8}\ ms^{-1}$

$\lambda = 3.3 \times 10^{-10}$ m

$= 3.3\ \text{Å}$ (Angstrom)

($1\ \text{Å} = 10^{-10}$ m)

(vi) $\lambda = h/p$

$p = \dfrac{6.63 \times 10^{-34}\ \text{J.s}}{3.3 \times 10^{-10}\ \text{m}} = 2.01 \times 10^{-24}$

(ii) $m = \dfrac{p}{v = c}$ $\quad p = 2.01 \times 10^{-24}$ m

$m = \dfrac{2.01 \times 10^{-24}}{3 \times 10^{8}\ \text{m/s}} = 6.697 \times 10^{-33}$ kg

(iii) $E = \dfrac{p^2}{2m}$

$= 3.01 \times 10^{-16}$ J

ASSIGNMENT :-

1) Calculate De Broglie wavelength associated with a proton moving with a velocity $1/20\ c$.

2) Find the energy of neutron in unit of electron volt which de broglie wavelength is 1Å ($M_n = 1.674 \times 10^{-27}$ kg)

3) Compute the de broglie wavelength of a proton whose K.E is equal to the rest-energy of an electron.

Solution

(i) $M_p = (1836 \times 9.1 \times 10^{-31})$ kg

$= 1.67 \times 10^{-27}$ kg

$\lambda = \dfrac{h}{M_p V} = \dfrac{6.63 \times 10^{-34}}{1.67 \times 10^{27} \times V}$

But, $v = 1/20 \times 3 \times 10^{8}$

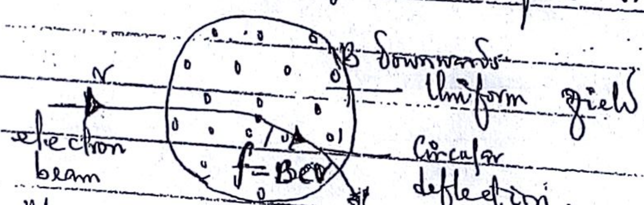
$= 1.5 \times 10^{8}$ m/s

$= \dfrac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 1.5 \times 10^{7}} = 2.647 \times 10^{-14}$ m

DEFLECTION IN A MAGNETIC FIELD

Consider an electron beam, moving with a speed v, which enters a uniform magnetic field of magnitude B acting perpendicular to the direction of motion, the force f on an electron is then given by BeV

The direction of the force is perpendicular to both B and V. Consequently, unlike the electric force, the magnetic force cannot change the energy of the electron. It deflects the electron but does not change its speed or kinetic Energy.



CIRCULAR MOTION IN UNIFORM MAGNETIC FIELD

The force BeV is always normal to the path of the beam. If BeV is large enough, the electron beam can turn round a complete circle. Then,

$$F = BeV = \frac{M_e V^2}{r}$$

velocity

(Since f = BeV will now be a centripetal force)

$$\therefore r = \frac{M_e V}{Be} = \frac{\text{momentum}}{Be}$$

EXAMPLE: Protons with a charge mass ratio of $1.0 \times 10^8 \, C\,kg^{-1}$ are rotated in a circular orbit of radius r when they enter a uniform magnetic field of 0.5T. Show that the number of revolutions per second f is independent of r and calculate f.

Solution

Suppose M_p is mass and e is the charge.

$$BeV = \frac{M_p V^2}{r}$$

$$r = \frac{m_p v}{Be}$$

$$= \frac{m_p}{e} \times \frac{v}{B}$$

Recall that, $v = r\omega$,

Relating, $v = \frac{Ber}{m_p} = r\omega$

$$\frac{Be}{m_p} = \omega \qquad \text{Since, } \omega = 2\pi f$$

$$\frac{Be}{m_p} = 2\pi f$$

$$\therefore f = \frac{Be}{m_p 2\pi} = \frac{B}{2\pi} \cdot \frac{e}{m_p}$$

This shows that f is independent of r.

$$f = \frac{0.5 \times 1 \times 10^8}{2 \times 3.14} = 8 \times 10^6 \text{ rev } s^{-1}$$

Example: An electron beam passes undeflected with uniform velocity v through two parallel plates, when a magnetic field of 0.01T is applied perpendicular to an electric field between the plates produced by a P.D of 100V. The separation of the plates is 5mm. Calculate v.

Solution

For no deflection,

$$Bev = Ee$$

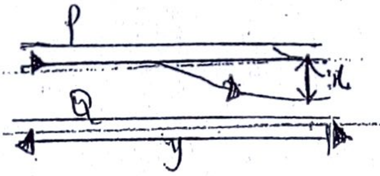
Recall that, $E = V/d$

then $Bev = \frac{V}{d} \cdot e$

$$v = \frac{V}{Bd} = \frac{100}{0.01 \times 5 \times 10^{-3} \text{ m}}$$

$$= 2 \times 10^6 \text{ m/s}$$

Quiz:



P, Q are plates with uniform electric field between them. An electron beam, moving parallel to P is deflected a distance x as shown after leaving the field. The length of the plates is y. If the length of the plates is increased to 3y and the separation of P and Q and the electric field strength are kept constant, the new deflection on leaving the field is ???

Solution

Recall that $x = ky^2$ from the question

$k = x/y^2$

if $y \to 3y$,

$x = k(3y)^2$

$= \frac{x}{y^2} \times 9y^2 = 9x$

The new x would be 9x.

(ii) An electron falls from rest through a potential difference of 100V. If the charge mass-ratio is 1.76×10^{11} Ckg^{-1}, what is
a) the velocity
b) the de broglie's wavelength.

Solution

$eV = \frac{1}{2} mv^2$

$2eV = mv^2$

$v = \sqrt{2\left(\frac{e}{m}\right)V} = \sqrt{2 \times 1.76 \times 10^{11} \times 100}$

$= 5.93 \times 10^6 \ ms^{-1}$.

b) $\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 5.93 \times 10^6} = 1.23 \ \text{Å}$

(iii) What PD is required in an electron microscope to give an electron a wave length of 0.5 Å.

Solution

Recall that,

$$\lambda = \frac{h}{mv}$$

$$v = h/m\lambda$$

Thus, $eV = k.e$

$$= \tfrac{1}{2} mv^2$$

$$= \tfrac{1}{2} m \left(\tfrac{h}{m\lambda}\right)^2$$

$$eV = \frac{1}{2} \times 9.1 \times 10^{-31} \left(\frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 5 \times 10^{-11}} \right)^2$$

$$eV = 0.5 \times 9.1 \times 10^{-31} \times 2.123 \times 10^{14}$$

$$V = \frac{0.5 \times 9.1 \times 10^{-31} \times 2.123 \times 10^{14}}{1.602 \times 10^{-19}} = 6.03 \times 10^{33}\ V$$

(iv) By Definition, a thermal neutron is a free neutron in a neutron gas at about 20°C. What is the K.E & λ of the neutron.

$(k = 1.38 \times 10^{-23}\ J/K)$

Solution

Recall that $K.E = \tfrac{3}{2} k_B T$

$$= \tfrac{3}{2} \times 1.38 \times 10^{-23} \times (293\ K)$$

$$= 6.0651 \times 10^{-21}\ J$$

$$\lambda = \frac{h}{\sqrt{2ME}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 6.07 \times 10^{-21}}}$$

$$= 0.147\ nm$$

RADIOACTIVITY CONT'D

$1u = 1\ amu$

$= 931\ MeV$

$E = MC^2$

$= \Delta m\, c^2$

mass defect

EXAMPLES:- What is the binding energy (in MeV) for oxygen $^{16}_{8}O$. atomic mass = 15.994915 u.

Solution

Oxygen Contains

8 neutrons = $^{1}_{0}n$ and

8 protons = $^{1}_{0}p$

$^{1}_{0}n$ = 1.0087 u,

$^{1}_{0}p$ = 1.0075 u

$\Delta m = [(8 \times 1.0087) + (8 \times 1.0087)] - 15.994915$

$= 0.137\ u$

Binding Energy = (0.137×931) MeV

= 127.6 MeV

03-06-2014

PHOTO- ELECTRICITY

- The emission of electrons from a metal surface when light or any other radiations of suitable wavelength incident on them is called Photoelectric effect.
- The photo-electric effect was discovered by Hertz in 1887.
- Einstein explained the photo electric effect on the basis of Planck's quantum theory of light (for which he received a nobel prize in 1921)
- EINSTEIN'S PHOTO ELECTRIC EQUATION.

Using the law of conservation of energy, and planck's quantum theory of light to explain the effect, Einstein assumed that one photon (hf) incident on the surface is used for two purposes.

- The energy of the photon hf, incident on the surface is used for two purposes

(i) A part of the energy is used for the liberation of electrons from the metal surface (W_o).

(ii) The remaining energy of the photon is used in giving kinetic energy to the emitted electron.

Thus, According to the law of conservation of Energy,

$$hf = W_0 + \frac{1}{2} mv^2$$

(work function)

$$\frac{1}{2} mv^2 = hf - W_0$$

Let f_0 be the minimum frequency of light called THRESHOLD FREQUENCY to start the photo electric effect, $W_0 = hf_0$

Then $\frac{1}{2} mv^2 = hf - hf_0$

$$\frac{1}{2} mv^2 = h(f - f_0)$$

where $h(f-f_0) \equiv V_s e$

stopping potential

f_0 = threshold frequency

f = photon / incident ray frequency.

LAWS OF PHOTOELECTRIC EMISSION

1. The no of photo electrons emitted per second (photo electric current) is directly proportional to the intensity of the incident radiation.
2. For each metal surface, there must be a certain minimum frequency of incident radiation to start photoelectric effect (f_0)
3. The maximum velocity of photo electron is independent of the intensity of incident radiation depends on the frequency of the incident radiation.
4. The photo electric emission is instantaneous process. The time lag between the incident of radiation and emission of photo electrons is very small (it is less than 10^{-9} second).

PHOTO ELECTRIC CELLS.

A device which converts light energy into electrical energy is called photo electric cell. They are mainly classified as

(i) Photo emissive cells

(ii) Photo voltaic cells

(iii) Photo conductive cells.

APPLICATIONS OF PHOTO CELLS

(i) They are used in the exposure device of cameras to determine the correct time exposure.

(ii) They are used for automatic opening and closing of door.

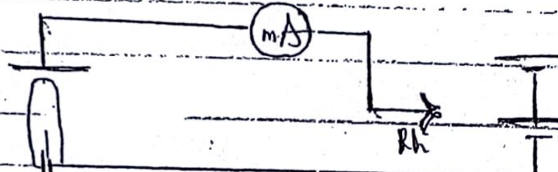
(iii) They are used in fire and burglar alarm.

(iv) They are used in space ships for harnessing solar energy.

(v) They are used in reproducing sound in cinematography.

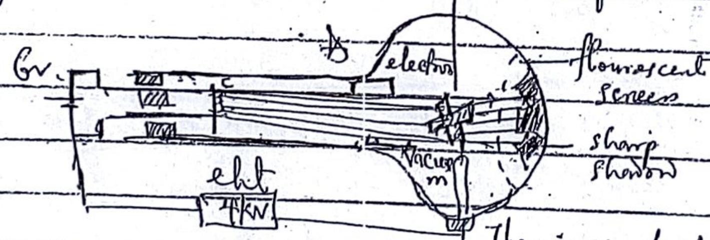
THERMIONIC EMISSION

Thermionic emission is the production of electron by a filament or metal when it is heated.



DIODE VALVE

Nowadays, a hot cathode is used to produce a supply of electrons.



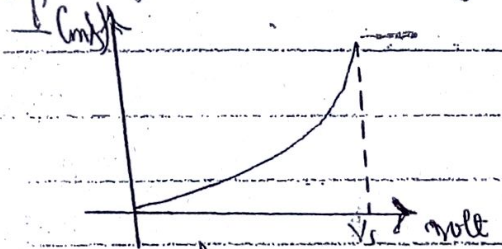
Thermionic diode or valve

Metals contain free electrons moving about rather like molecules in a gas. If the temperature of the metal is raised, the thermal velocity of the electron will be increased. The chance of electron escaping from the attraction of the +ve ions, fixed in the lattice, will then also be raised.

Thus, by heating a metal such as tungsten to a high temperature, electrons can be "boiled off". This is called Thermionic Emission. Fast-moving electrons emitted from a heated cathode C produce a sharp shadow of a maltese cross on the flourescent screen. Thus, electrons travel in straight lines, and (2) they also produce heat when incident on a metal — a fine piece of platinum glows, for example.

DIODE VALVE

Considering the diode valve drawn earlier, by plotting the current produced against the voltage reading, the graph below is obtained.



where V_s is the stopping potential

DIODE CHARACTERISTICS

At a point, a constant current is observed due to the saturation of electrons at the anode. To continue the flow of current, a second metal is introduced between the anode and the filament. By doing this, a diode valve has been created. The graph obtained is called a diode characteristic graph.

To produce a DC current an AC current has to be rectified. A rectifier is employed. An example of a rectifier is a diode valve.

=> Diode valves are used in photocopying machines.

=> The most common function of a diode is to allow an electric current to pass in one direction (called the diode's forward direction), while blocking current in the opposite direction (the reverse direction).

Thus, the diode can be viewed as a rectifier.

In summary, in electronics, a diode is a two-terminal electronic computer with assymetric conductance. A semiconductor diode (is the most common type today) is a crystalline piece of semiconductor material with a p-n junction.

FUNCTIONS OF DIODES.

=> For Rectification (act as rectifier to convert AC current to DC current including the extraction of modulation from radio signals in radio recievers.

Also, semiconductor diode's current - voltage characteristic can be tailored by varying the semiconductor materials and doping (introducing impurities into the materials).

These are exploited in special-purpose diodes that perform many different functions. Diodes use to

(i) regulate voltage (Zener diodes)
(ii) Protect circuits from high voltage surges (avalanche diode)
(iii) electrically tune radio and T.V recievers (Varactor diodes)
(iv) generate radio frequency oscillations (tunnel diodes, Gunn diodes, IMPATT diodes.
(v) Produce light (Light emitting diodes L.E.D)

NOTE: Tunnel diodes exhibits negative resistance, which makes them useful in some types of circuits.

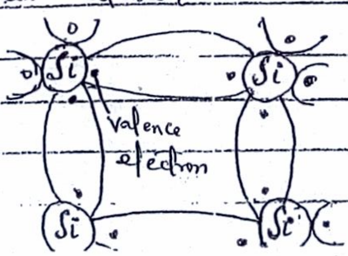
SEMI- CONDUCTORS

A solid is said to be a semi-conductor if it conducts heat & electricity when it is slightly heated above room temperature or when it is doped (i.e impurities are added to it).

Semi-conductors are a class of materials which have a resistivity about ten million times higher than that of a good conductor such as Copper — 2nd best, aluminium — 3rd best or silver — 1st best.

Examples of semi-conductors are silicon, germanium, carbon. They are bonded by covalent bonds (i.e one valence atom electron is shared with each of four surrounding atoms in a tetrahedral arrangement forming covalent bonds). Carbon, however cannot be used becos of its hard nature.

The diagram below is a two dimensional diagram of the structure, showing four silicon atoms, each having four valence electrons around them.



Due to the movement of a valence electron from atom to atom, holes spread throughout the Semi-conductor. Since an electron carries a negative charge -e, a hole, moving in the opposite direction to the electron is equivalent to a positive charge +e. So, moving holes are equivalent to moving positive charges

In Summary,

1. In Semiconductors, then there are two kinds of charge carriers: a free electron (-e) and a hole (+e). In contrast: a metal (copper), a good conductor, has only one kind of charge carrier, the free electron.
2. The escape of a valence (bound) electron from an atom produces electron-hole.

Pair of Charge Carriers

A Semi conductor which conducts when slightly heated is said to be pure (Intrinsic) i.e pure or Intrinsic Semi-Conductors has charge carriers which are thermally generated.

Also, if a Semi-Conductor conducts when doped (Impurities are added) in it, it's said to be an Impure or extrinsic Semi-Conductor.

=> Unlike the pure intrinsic conductors, the Impure Semi-conductor is widely used in the electronics industry.

In Summary.

In a pure (Intrinsic) Semi Conductors;

i. There are two kinds of charge carriers, holes (+e) and electrons (-e)
ii. The number of holes = the number of electrons.
iii. The drift velocity of the electrons is greater than that of the holes.

P-SEMICONDUCTORS

P-Semiconductors are made by adding foreign atoms which are trivalent to pure germanium or Silicon. Examples are boron or Indium (group III). When an element from group III is used for doping, it

Completes the trivalent lattice site thereby leaving a hole behind. In this way, an enormous increase in the number of holes making the Semi-conductor positively charged. Hence, a p-type Semi conductor results.

In a P-Semiconductor, the impurity atoms are called acceptors. In this case because each "accepts" an electron when the atom is introduced into the crystal.

N- SEMICONDUCTORS

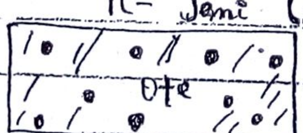
N-Semiconductors are made by adding group V elements (e.g Arsenic) to pure germanium or silicon. When a group V element is used for doping, it totally fills up the tetravalent lattice points of the Semi-conductor. The fifth electron may thus become free to wonder through the crystal..

In this way, the enormous increase in the number of electrons makes the Semiconductor negatively charged. In a N-semiconductor, the impurity atoms are called the donors. In this case, because they donate electrons as carriers.

Thus (In Summary) In an N-semiconductor, conduction is due mainly to the negative charges or electrons (which are the majority carrier or dominant species), with +ve charges (holes) as minority carriers.

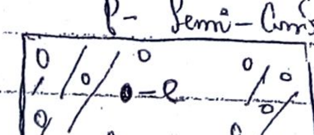
In a P-Semiconductor, conduction is due mainly to +ve charges or holes (which are the majority carriers), with -ve charges (electrons) as minority carriers.

n- Semi Conductor.



majority electrons (-e)
minority holes (+e)

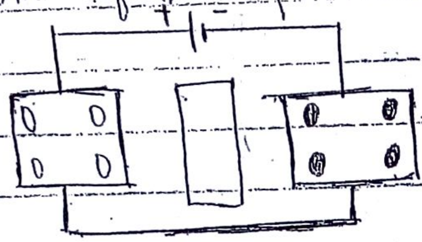
P- Semi-Conductor



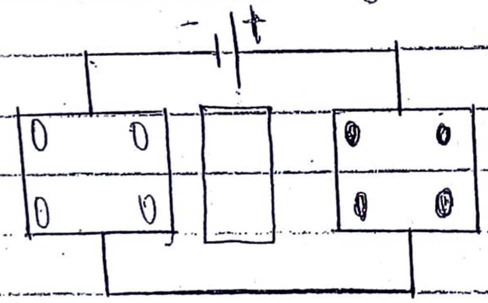
majority holes (+e)
minority holes (-e)

P-N JUNCTION

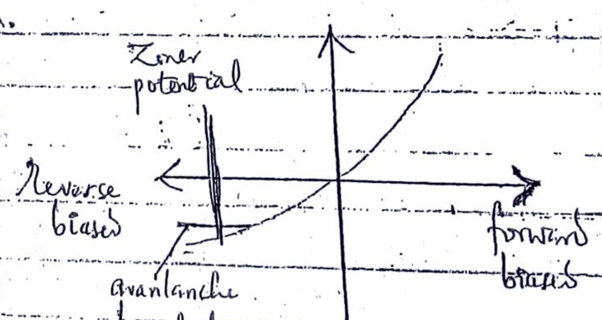
By a special manufacturing process, P- and n- semiconductors can be united so that a boundary or junction is formed between them. It is called a p-n junction. This junction is extremely thin and of the order 10^3 mm. The narrow region or layer at the p-n junction which contains the negative and positive charges is called the "Depletion layer". The width of the depletion layer is of the order 10^3 mm.



The diagram is forward biased (or forward potential) because the negative terminal is connected to the n-junction and the positive terminal to the P-junction.



The diagram is reverse biased (Reverse potential) because the positive terminal is connected to the n-junction and the negative terminal to the P-junction.

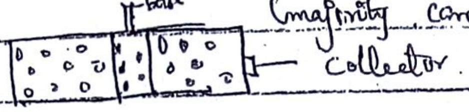


P-N JUNCTION CHARACTERISTIC

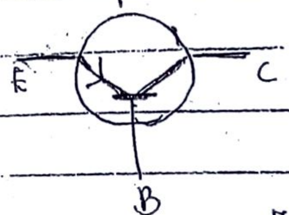
TRANSISTORS

The transistor is a current amplifier — A transistor is made from three layers of P- and n- semiconductors. They are called respectively, the emitter (E), base (B) and collector (C)

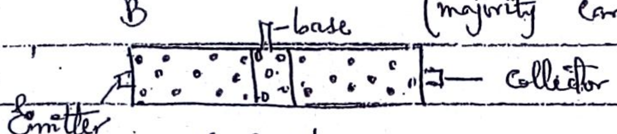
(majority carriers are holes)



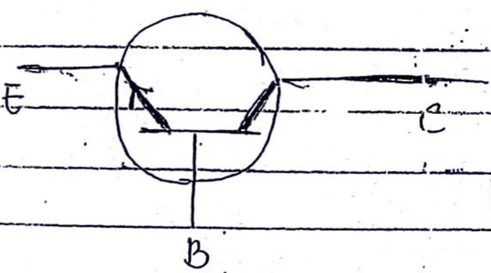
P-N-P



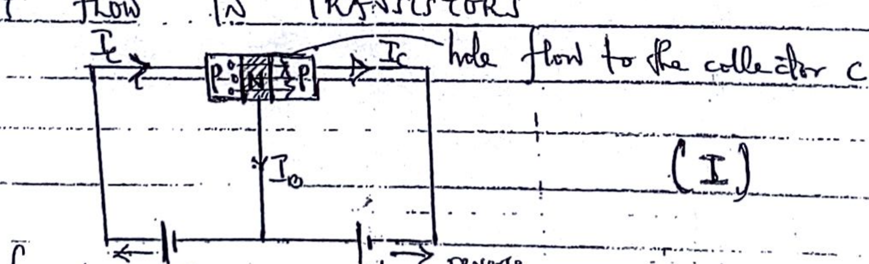
(majority carrier are electrons)



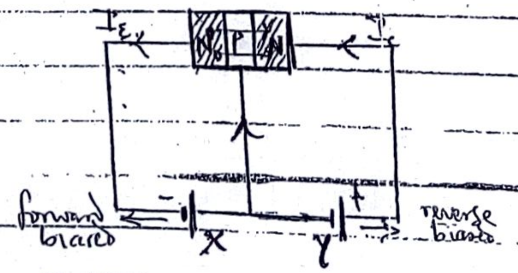
N-P-N



CURRENT FLOW IN TRANSISTORS



(I)



(II)

This is called the common-base mode of using a transistor.
Note carefully the polarities of the two batteries.
In (I), the positive pole of the supply voltage X is joined to the Emitter E but the negative pole of the supply voltage Y is joined to the collector C.
In the case of II, (N-P-N) transistor, the negative pole of one battery is joined to the emitter and the +ve pole of the other is joined to the collector. In this way, the emitter-base is forward biased and the collector base is reverse biased.
From Kirchhoff's first law, it follows that always, if I_E is the emitter current, $I_E = I_C + I_B$.
Although the action of n-p-n transistors are similar in principle to p-n-p transistors the carriers of the current in the N-P-N transistors, are mainly electrons but holes in the p-n-p transistors.
Electrons are more ~~speedy~~ carriers than holes, so n-p-n transistors are used in high frequency and computer circuits where the carriers are required to respond very quickly to signals.

THE HYDROGEN ATOM

- It has a diameter of about 0.1nm; it consists of a proton as the nucleus (with a radius of about 10^{-15}m) and a single electron.

ELECTRON ORBITS

The first effective model of the atom was introduced by Niels Bohr in 1913. For electron's de Broglie wave to fit in an orbit of radius r, the following must be true

$$mv_n r_n = \frac{nh}{2\pi}$$

The centripetal force that holds the electron in orbit is supplied by Coulomb attraction between the nucleus and the electron. Hence $F = \frac{ke^2}{r^2} = \frac{mv_n^2}{r_n}$

→ The energy of an atom when it is in the nth state is

$$E_n = \frac{-13.6}{n^2} eV$$

For a nucleus with charge Ze orbited by a single electron the corresponding relations are

$$r_n = (0.053 nm)\left(\frac{n^2}{Z}\right) \text{ and}$$

$$E_n = \frac{-13.6Z^2}{n^2} eV$$

Where Z is called the atomic number.

→ The zero of energy is taken to be the ionized atom (i.e. infinite orbital radius) at $n = \infty$, $E = 0$

EMISSION OF LIGHT

When an isolated atom falls from one energy level to a lower one, a photon is emitted. The λ and f of the photon is given by

$$hf = \frac{hc}{\lambda} = \text{energy lost by the system.}$$

At ground state, n=1 thus

$E = -13.6eV$ from $E_n = 13.6/n^2$ eV.

It is convenient to remember that a 1240nm photon has an energy of 1eV and that photon energy varies inversely with λ.

SPECTRAL LINES

Lyman $1/\lambda = R(1/1^2 - 1/n^2)$, $n = 2, 3, ...$

Balmer, $1/\lambda = R(1/2^2 - 1/n^2)$ $n = 3, 4, ...$

Paschen, $1/\lambda = R(1/3^2 - 1/n^2)$ $n = 4, 5, ...$

Where $R = 1.0974 \times 10^7 m^{-1}$.

EXAMPLE; What λ does a hydrogen atom emit as its excited electron falls from the n=5 state to n=2 state? 3(s-f)

[illegible]

$E_n = \frac{-13.6}{n^2}$ eV

$E_6 = -0.54$ eV $E_2 = -3.40$ eV

Energy diff $= 3.40 - 0.54$

$= 2.86$ eV

Recall that $1eV \propto \frac{1}{1240nm}$

1eV corresponds to 1240nm in inverse proportion.

$\lambda = \left(\frac{1.00eV}{2.86eV}\right)(1240\,nm)$

$= 434$ nm

Thus, the higher the energy, the lower the photon λ.

EXAMPLE: The series limit λ of the Balmer series is emitted as the electron in the hydrogen atom falls from the $n = \infty$ state to the $n = 2$ state. What is the wavelength of this line in (3 s.f.)

Solution

$\Delta E = 3.40 - 0$

$= 3.40$ eV

$\lambda = \frac{hc}{\Delta E} = 365$ nm.

EXAMPLE: One spectral line in the hydrogen spectrum has a wavelength of 821nm. What is the energy difference between the two states that gives rise to this line?

Solution

$\Delta E = hc/\lambda$

$\Delta E = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{821 \times 10^{-9}\,m}$

$= 2.423 \times 10^{-19}$ J

$= 1.51$ eV

BOHR'S MODEL

The force F is provided by the electrical attraction between two charges each with magnitude e,

$$F = \frac{ke^2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^2}$$

Thus, $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r_n^2} = \frac{mV_n^2}{r_n} \text{ ------ } *$$

Recall that,
angular momentum L,
$$= mV_n r_n = \frac{nh}{2\pi}$$

Then $$V_n = \frac{nh}{mr_n 2\pi}$$

$$V_n^2 = \frac{n^2h^2}{m^2r_n^2\,4\pi^2}$$

Also, from above $(*)$
$$V_n^2 = \frac{e^2}{4\pi\varepsilon_0 m r_n}$$

$$V_n^2 = \frac{e^2}{4\pi\varepsilon_0 m r_n} = \frac{n^2h^2}{m^2r_n^2\,4\pi^2}$$

$$\frac{e^2}{\varepsilon_0} = \frac{n^2h^2}{mr_n\pi}$$

$$r_n = \varepsilon_0\frac{n^2h^2}{\pi me^2} \text{ ------ } (i)$$

If we substitute for r_n in
$$mV_n r_n = \frac{nh}{2\pi}$$

$$mV_n\left(\frac{\varepsilon_0 n^2h^2}{\pi me^2}\right) = \frac{nh}{2\pi}$$

$$V_n = \frac{e^2}{\varepsilon_0 2nh} \text{ ------ } (ii)$$

Eqn ① Shows that the orbit radius r_n is proportional to n^2. The smallest orbit r_1 corresponds to $n=1$. This minimum (smallest) radius is called the Bohr's radius.

$\equiv 0.529\ A^\circ$

$= 0.529 \times 10^{-10}\ m$

$\equiv 5.29\ nm$

Thus, $r_n = n^2 r$

The permitted, non radiating orbits have radii r_1, $4r_1$, $9r_1$ and so on. The value of n for each orbit is called the principal quantum number for that orbit.

ENERGY LEVELS

We can use eqn ⓘ & ⓘⓘ to find the kinetic and potential energies E_{kn} and E_{pn} for an electron in the orbit with quantum number n.

$$_nE_k = \frac{1}{2} m v_n^2$$

$$= \frac{1}{2} m \left(\frac{e^2}{\varepsilon_0 2nh} \right)^2$$

$$= \frac{me^4}{8\varepsilon_0^2 n^2 h^2}$$

For potential Energy,

Recall that $\frac{-ke^2}{r} = \frac{-1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r_n}$

$$= \frac{-me^4}{4\varepsilon_0^2 n^2 h^2}$$

$$T_{energy} = E_k + E_p$$

$$= \frac{me^4}{8\varepsilon_0^2 n^2 h^2} + \left[-\frac{me^4}{4\varepsilon_0^2 n^2 h^2} \right]$$

$$= -\frac{me^4}{8\varepsilon_0^2 n^2 h^2}$$

NOTE: The negative sign shows we have taken the reference level of potential energy to be zero when the electron is at rest

at an infinite distance from the nucleus.
Thus for an emission from n_1 to n_2.

$$\Delta E = hf = E_2 - E_1$$

$$= \frac{me^4}{8\varepsilon_0^2 n^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{hc}{\lambda} = \frac{me^4}{8\varepsilon_0^2 n^2 h^2}$$

$$\frac{1}{\lambda} = \frac{me^4}{8\varepsilon_0^2 h^3 c}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where Rydberg's constant

$$= \frac{me^4}{8\varepsilon_0^2 ch^3} = 1.0974 \times 10^7\ m^{-1}$$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$\frac{1}{\lambda} \equiv$ Wave no.

QUIZ :: According to the Bohr's model of the hydrogen atom, as we look at higher values of n, the wavelength of the photon emitted due to transitions between adjacent electron orbits.

A) gets progressively smaller.
B) gets progressively larger.
C) remains constant

ANSWER is B

Recall that from

$$E_n = \frac{-13.6}{n^2}\ eV$$

The higher the value of n, the smaller the Energy.

Thus in $E_n = \frac{hc}{\lambda}$

the smaller the E_n means a larger λ.
Therefore when n increases, E_n decreases and thus λ increases.
PROOF

$$\frac{me^4}{8\varepsilon_0^2 ch^3}$$

$$= \frac{9.11 \times 10^{-31} \times (1.6 \times 10^{-19})^4}{8 \times (8.85 \times 10^{-12})^2 \times 3 \times 10^8 \times (6.63 \times 10^{-34})^3}$$

$$= \frac{59.7033 \times 10^{-107}}{5.47820 \times 10^{-113}}$$

$$= \frac{10.8983 \times 10^6}{1}$$

$$= 1.08983 \times 10^7 \, m^{-1}$$

$$\simeq 1.09 \times 10^7 \, m^{-1}$$

Relation of the Rydberg Constant to fundamental Constants

$$nCR = \frac{1}{\varepsilon_0^2 8h^2} Me^4 = 13.6 \, eV$$

Comparing with $\boxed{E_n = \frac{-hcR}{n^2}}$

PROOF : if we multiply $1/\lambda = R\left(1/2^2 - 1/n^2\right)$ by hc to get E, we get

$$E = \frac{hc}{\lambda} = hcR\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

$$= \frac{hcR}{2^2} - \frac{hcR}{n^2} \quad \text{------ (iii)}$$

Looking at $\Delta E = E_i - E_f$

$$hf = E_i - E_f \quad \text{------ (iv)}$$

Eqn (iv) & (iii) will agree if

$$E_n = \frac{hcR}{n^2}$$

The Balmer Series (as well as others that we will mention shortly) therefore suggests that the hydrogen atom has a series of energy levels, which we will denote as E_n

$$\frac{-hcR}{n^2} = -\frac{13.6}{n^2} \, eV \qquad n = 2, 3, 4, \ldots$$

This energy are -ve bcos we have arbitrarily chosen the potential energy to be zero at very large values of r.

BOHR MODEL OF THE ATOM (cont'd)

$$F = \frac{ke^2}{r} = \frac{\mu v^2}{r} \quad ---- (1)$$

NOTE: the notation of μ for the mass is in order. Up to now we have been using the letter m, for the mass of the electron. However, the electron in a hydrogen atom is not moving around the proton, instead, both are moving around a common centre of mass. We can incorporate this effect by introducing the reduced mass μ as $\mu = \frac{mM}{m+M}$ where $m = 9.10938 \times 10^{-31}$ kg and $M = 1.67 \times 10^{-27}$ kg (proton)

This reduced mass has n numerical value.

$\mu = 9.10442 \times 10^{-31}$ kg because the proton mass is a factor of 1836 times bigger than electron mass, the term $M/(m+M)$ is very close to 1, and thus $\mu \simeq m$ to 1 part in 2000.

However, we will use the reduced mass μ in this case. The advantage is that historically, people have introduced a quantum no. m in the solution to the hydrogen problem, and we want to avoid confusion.

QUANTIZATION OF ORBITAL ANGULAR MOMENTUM

Angular momentum vector of a point particle as

$$\vec{L} = \vec{r} \times \vec{p}$$

Recall, $\vec{p} = mv$

In this case, μv

$$\vec{L} = \vec{r}\,\mu v = n\hbar \quad ----- *$$

Let's first work out the consequences. To begin calculation for

The Bohr model of the hydrogen atom, start with eqn (i)
multiply both sides by μr

$$\frac{Ke^2}{r^2} = \frac{\mu v^2}{r} \Rightarrow r\mu ke^2 = \mu^2 v^2 r$$

Looking closely, the R.H.S is now the square of the angular momentum $\vec{L} = \mu v r$

Using eqn (ii)

$$r = \mu ke^2 = \mu^2 v^2 r^2 = n^2 \hbar^2$$

Solving for r

$$r = \sqrt{\frac{\hbar^2}{\mu^2 v^2} n^2} = \frac{\hbar^2}{\mu ke^2} n^2$$

$$\equiv a_0 n^2$$

This equation gives us the allowed radii in the Bohr model of the hydrogen atom. They are ~~perpendicular~~ proportional to the square of the quantum no n, and the proportionality constant a_0 is called the Bohr radius $a_0 = \frac{\hbar^2}{\mu ke^2}$

$$= \frac{(1.05457 \times 10^{-34}\ J \cdot s)^2}{(9.1 \times 10^{-31}\ kg)(8.98 \times 10^9)(1.6 \times 10^{-19})}$$

$$a_0 = 5.295 \times 10^{-11}\ m$$

$$= 0.05295\ nm$$

$$= 0.5295\ A^\circ$$

(iii) putting the value of r in (ii)

$$r\mu ke^2 = \mu^2 v^2 r^2$$

$$v^2 = \frac{ke^2}{\mu r}$$

$$v = \sqrt{\frac{ke^2}{\mu r}} \quad \left(\text{Recall } r = a_0 n^2\right)$$

$$= \sqrt{\frac{ke^2}{\mu a_0 n^2}} \text{ - - - - - - (iv)}$$

Thus, $v = \frac{1}{n} 2.188 \times 10^6 \, ms^{-1}$

$= \frac{1}{n} 0.007297 c$

This speed is 0.73% of the speed of light for n=1 and falls monotonically for the higher orbits.

Therefore, the total energy of the electron in orbit is the sum of its potential and kinetic Energies.

$$E = \tfrac{1}{2} \mu v^2 - \frac{ke^2}{r}$$

$$= -\tfrac{1}{2} k e^2/r = -\tfrac{1}{2} k e^2/a_0 n^2$$

$$= -E_0 \, 1/n^2 \quad \text{----} \; (v)$$

Where $E_0 = \frac{ke^2}{2a_0} = 13.6 eV$

In many textbooks, the Bohr radius is defined in terms of the mass of electron m_e, rather than the reduced mass μ of the electron in the hydrogen atom.

$$a_0 m_e = \frac{\hbar^2}{m_e k e^2}$$

It has the value 5.292×10^{-11} using this definition.

SPECTRAL LINES IN THE BOHR MODEL

When an electron in a higher energy state with quantum no n_2 "jump" to a lower energy n_1, it would emit a photon with energy equal to the difference in the energies between the two states.

$$E = hf \; : \; E_{n_2} = E_{n_1} + hf$$

$$E_{n_2} = E_{n_1} + hc/\lambda$$

Using the eqn (v)

$$-\frac{ke^2}{2a_0} \frac{1}{n_2^2} = -\frac{ke^2}{2a_0} \cdot \frac{1}{n_1^2} + \frac{hc}{\lambda}$$

$$\Rightarrow \frac{1}{\lambda} = \frac{ke^2}{2hca_0} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$\frac{ke^2}{2hca_0} = \text{Rydberg Constant} = 1.097\times10^7\,m^{-1}$$

Given $a_0 = \frac{h^2}{4\pi^2 mke^2}$

$$R_H = \frac{ke^2}{2hc\left(h^2/4\pi^2 mke^2\right)} = \frac{2\pi^2 k^2 e^4 m}{4\pi c h^3}$$

MILLIKAN'S EXPERIMENT

- In 1909, Millikan started a series of experiments lasting many years which supplied evidence for the atomic nature of electricity and provided a value for the magnitude of electronic charge.
- For the magnitude of electronic charge, the principle of his method is to observe very small oil drops, charged either positively or negatively, falling in air under gravity and then either rising or being held stationary by electric fields.
- For a spherical drop of radius r, moving with uniform velocity v, through a homogeneous medium having co-efficient of viscosity η; Stoke's law states that the viscous force retarding its motion is $6\pi\eta r v$.

In falling, the drop attains its terminal velocity almost at once bcos it is so small. The retarding force acting up the medium equals its weight given by mg.

(where m = volume × density) $= \frac{4}{3}\pi r^3 \times \rho$

The force $= \frac{4}{3}\pi r^3 \rho g$

(ρ is the density of the oil and g is the acceleration due to gravity.)

F_r = viscous force

motion of oil drop.

velocity $= v$

F_g = Gravitational force − Bouyant force

When the oil drop with constant velocity, it means that there should be no force acting on it.

meaning that : $f_v = f_g$

$f_v = 6\pi\eta r v_1$ ---------- (i)

$f_g = \frac{4}{3}\pi r^3 \rho g$ ---------- (ii)

$\therefore 6\pi\eta r v_1 = \frac{4}{3}\pi r^3 \rho g$ ---------- (iii)

and $\boxed{r^2 = \frac{9\eta v_1}{2g\rho}}$ ---------- (iv)

where r = radius of oil drop
η = Coefficient of viscosity
v_1 = velocity of the oil drop
ρ = density on oil.

eqn (iv) can be re-written as

$$r^2 = \frac{9\eta v_1}{2g(\rho_{oil} - \rho_{air})}$$

ρ_{air}

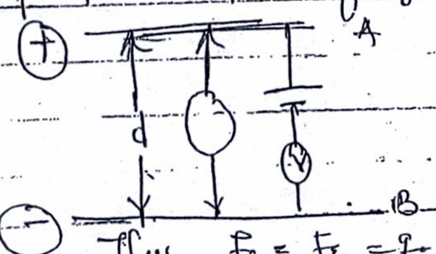
where, ρ_{oil} = Density of oil
ρ_{air} = Density of air / medium

— Suppose the drop under observation has a -ve charge q. When a PD is applied to the plates so that the top one is +ve, an electric field E is created which exerts an upward force on the top drop at rest, then the electric force experienced by it is Eq (since $E = F/q$).

Here, F_E = electric force $= qE$

This electric force can do two things

(i) Stop the motion of the oil drop



$F_E = q \cdot E$.

Thus, $F_g = F_E = q \cdot E$

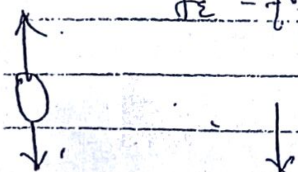
$$F_q = q \times V/d$$
$$= \frac{q \times f_g \times d}{V}$$
$$q = \frac{4\pi r^3 \rho d}{3V}$$

(iii) Recall $f_q = f_r$, also
$$q = \frac{6\pi\eta r v_1 d}{V}$$

(2) The oil drop may rise up with constant velocity V_2 which may be experimentally calculated by recording the time required for the oil drop to rise a certain distance.

NOTE:- Now the viscous force acts downward bcs the oil drop is moving up.

$$F_E = q \cdot E = q \cdot V/d$$



motion of oil drop
velocity $= V_2$

$$F_g = (6\pi\eta r v_1) \quad F_v = 6\pi\eta r V_2$$
$$= \frac{4}{3}\pi r^3 \rho g$$

These forces should balance each other.

$$F_E = F_g + F_v$$
$$q\,V/d = 6\pi\eta v_1 r + 6\pi\eta V_2 r$$
$$q = \frac{6\pi\eta r(V_1+V_2) \times d}{V}$$

Millikan found that the charge on an oil drop, whether +ve or -ve was always an integral multiple of a basic charge.

He studied drops having charges many times having this basic charge and he was able to change the charge on a drop. The same minimum charge equal to that of on a monovalent ion was always involved. The value of the atom of electric charge i.e the e

Electronic charge e is 1.6×10^{-19} C
- From J.J Thompson's experiment on cathode ray tube, charge to mass ratio $= e/m = 1.76 \times 10^{11}$ C/kg
and $m = e/(e/m)$

$$= \frac{1.602 \times 10^{-19}}{1.76 \times 10^{11}} = 9.1 \times 10^{-31} \text{ kg}$$

EXAMPLE:

Calculate the radius of a drop of oil with density 900 kg m^{-3}, which falls with a terminal velocity of 2.9×10^{-2} cms^{-1} through air of viscosity 1.8×10^{-5} Nsm^{-2} (Ignore the density of air). If the charge on the oil drop is $-3e$, what PD must be applied between two plates 5cm apart for the drop to be held stationary between them?
($e = 1.6 \times 10^{-19}$ C and $g = 10$ m/s^2)

Solution

$\eta = 1.8 \times 10^{-5}$ Nsm^{-2}

$\rho = 900$ kg m^{-3}

$v = 2.9 \times 10^{-2}$ Cms^{-1}

$= 2.9 \times 10^{-4}$ ms^{-1}

$g = 10$ m/s^2.

Then,

$$r^2 = \frac{9\eta v_1}{2g\rho}$$

$$r = \sqrt{\frac{9 \times 1.8 \times 10^{-5} \times 2.9 \times 10^{-4}}{2 \times 10 \times 900}}$$

$= 1.6 \times 10^{-6}$ m

$= 1.6 \times 10^{-4}$ m

Recall that

$$F_E = qE$$
$$= q \times V/d$$

if $q = 3e$ (given in the question)

Since $F_E = F_g$

$$qE = \tfrac{4}{3}\pi r^3 \rho$$

$E \times 3e = \frac{4}{3}\pi r^3 \rho$

$E = \frac{4\pi r^3 \rho d}{qe}$

$V = \frac{4\pi \times (1.6 \times 10^{-6})^3 \times 900 \times 5 \times 10^{-2}}{9(1.6 \times 10^{-19})}$

$V_f = 1600V$

EXAMPLE:- An oil drop of density 350 kgm-3 and radius 2.04 mm falls with a constant velocity of 0.3 m/s.

(i) Find the viscosity of air in the chamber. (the density of air = 1.29 kgm-3).

(ii) Suppose the charge on the oil drop is +16e. If a PD of 155000mV is applied between the plates which are 0.2 cm apart, find the velocity at which the oil drop will move before it attains a stationary state.

Solution

$\rho = 350 \, kg m^{-3}$ $\qquad V_1 = 0.3 m/s$

$r = 2.04 \, mm$ $\qquad \eta = ??$

$= 2.04 \times 10^{-3} m$

(i) Recall that,

$r^2 = \frac{9\eta v_1}{2g\rho}$

$= \frac{9\eta v_1}{2g(\rho_{oil} - \rho_{air})}$ $\quad = \frac{r^2[2g(\rho_{oil} - \rho_{air})]}{9v_1}$

$= \frac{(2.04 \times 10^{-3})(2 \times 10)(350 - 1.29)}{9 \times 0.3}$ $\quad = 1.07 \times 10^{-2} \, NJm^{-2}$.

(ii) If a Pd is applied, $F_E = F_q + F_v$

$qE = 6\pi\eta v_1 r + 6\pi\eta v_2 r$

$q \cdot V/d = 6\pi\eta r(v_1 + v_2)$

$v_1 + v_2 = \frac{qv}{\ldots}$

$$V_2 = \frac{qr}{d(6\pi \eta r)} - V_1$$

$$= \frac{16(1.6 \times 10^{-19}) \times 155000000000}{2 \times 10^{-3}(6\pi \times 1.07 \times 10^{-2} \times 0.000204)} - 0.3$$

$$= 0.482 - 0.3$$

$$V_2 = 0.182 \text{ m/s}$$

X-RAYS

- X-rays so named because their nature was at first unknown, & were discovered in 1895 by Röntgen.
- They are produced whenever cathode rays (electrons) are brought to rest by matter.

PRODUCTION

A modern x-ray tube is highly evacuated and contains an anode and a tungsten filament connected to a cathode as shown in the diagram.

Electrons are obtained from the filament by thermionic emission and are accelerated to the anode by a PD, typically up to 100kV.

- The anode is a copper block inclined to the electron stream and having a small target of tungsten, or another high-melting point metal, on which electrons are focused by the concave cathode.
- The tube has a lead shield with a small window to allow the passage of the x-ray beam.

N.B; Less than ½ percent (0.5%) of the kinetic Energy of the electrons is converted into x-rays. The rest of the kinetic energy becomes internal energy of the anode which has to be kept cool by circulatory oil or water through channels in it or by the use of cooling fins.

(ii) The intensity of the x-ray beam increases when the number of electrons hitting the target increases and this controlled by the with the P.D across the X-ray tube, an electron of charge e has work, eV done on it by the electric field and so,

$eV = \frac{1}{2} mv^2$ ——— (2)

Equalling (i) & (ii) $hf = eV$ ——— (3)

the value of f in eqn (3) is the maximum frequency of the X-rays emitted at PD V_B, since all the energy of the electron is converted to photon. the corresponding wavelength will have a minimum value, and if this is λ_{min} then,

$C = f\lambda_{min}$ ——— (4) where c is the speed of X-rays $= 3 \times 10^8$ m/s, $\lambda_{min} = hc/eV$ ——— (5)

as V increases, we see from (5) that λ_{min} decreases, which means that X-rays of higher frequency and greater penetrating power are emitted.

The electrical power input in the X-ray tube is given by

Power Input = IV ——— (6)

the current I, through the tube is given by $I = ne/n$ ——— (7)

where n is the no of electrons striking the target per second and e is the electronic charge (1.6×10^{-19}C).

EXAMPLE: An X-ray tube operates at 30kV and the current through it is 2.0mA. calculate;

(i) the electronic power input

(ii) the number of electrons striking the target per second.

(iii) the speed of the electrons when they hit the target and

(iv) the lower wavelength limit of the X-rays emitted.

Solution.

(i) $V = 30$ kV

$= 30 \times 10^3$ V

$I = 2.0$ mA

$= 2 \times 10^{-3}$ A

Power Input = IV

$= 2 \times 10^{-3} \times 30 \times 10^3 = 60$ ...

(ii) $I = ne$

$\therefore n = I/e$

$= \frac{2 \times 10^{-3}}{1.6 \times 10^{-19}}$

$= 1.25 \times 10^{16}\ Cs^{-1}$

(iii) Recall that,

$hf = \frac{1}{2} mv^2 = eV$

then,

$\frac{1}{2} mv^2 = eV$

$v = \sqrt{\frac{2eV}{m}}$

$= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 30000}{9.1 \times 10^{-31}}} = 1.03 \times 10^{8}\ ms^{-1}$

(iv) Recall that, $hf = \frac{hc}{\lambda_{min}} = eV$

Thus, $\lambda_{min} = hc/eV$

$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times 30000} = 0.0414nm.$

ASSIGNMENT

(1) The P.D between the target and cathode of an X-Ray tube is 20kV and the current is 20 mA, only 0.5% of the total energy supplied is emitted as X-rays.

a) What is the minimum wavelength of the emitted x-rays?

b) At what rate must heat be removed from the target in order to keep it at a steady temperature.

2) What is the potential difference between the target and cathode when the spectrum of X-rays are produced? The minimum wavelength of the x-rays produced is 6.19×10^{-11} m. If the total energy of the bombarding electrons is only 0.2% of changed into x-rays and the x-ray tube ...