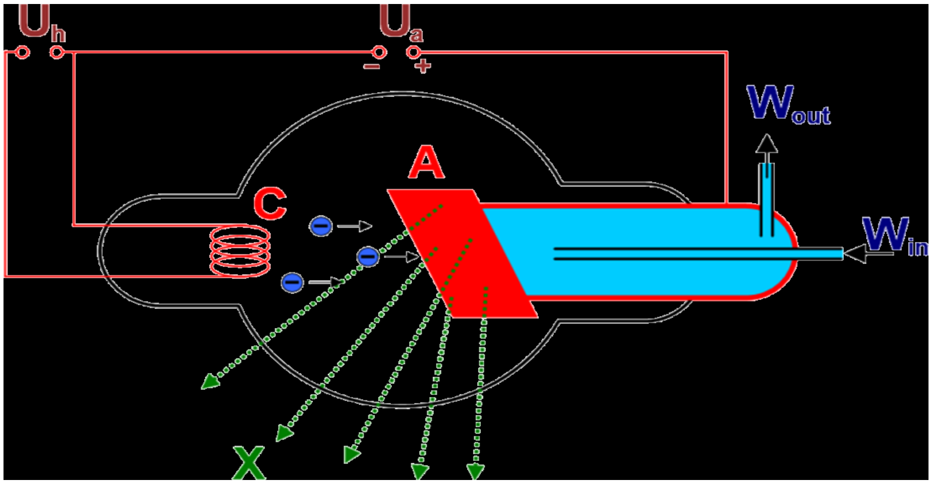
PHY 142 Lecture Note 2014/2015

X -RAYS

Discovery of X-rays

Discovered in late 1895 by a German physicist, W. C. Roentgen was working with a cathode ray tube in his laboratory. **Production of X-rays**

An X-ray tube is a vacuum tube designed to produce X-ray photons. The Crookes tube is also called a discharge tube or cold cathode tube. A schematic x-ray tube is shown below.



The glass tube is evacuated to a pressure of air, of about 100 pascals, recall that atmospheric pressure is 106 pascals. The anode is a thick metallic target; it is so made in order to quickly dissipate thermal energy that results from bombardment with the cathode rays. A high voltage, between 30 to 150 kV, is applied between the electrodes; this induces an ionization of the residual air, and thus a beam of electrons from the cathode to the anode ensues. When these electrons hit the target, they are slowed down, producing the X-rays. The X-ray photon-generating effect is generally called the Bremsstrahlung effect, a contraction of the German "brems" for braking, and "strahlung" for radiation. The radiation energy from an X-ray tube consists of discrete energies constituting a line spectrum and a continuous spectrum providing the background to the line spectrum.

Properties of X-rays

i. X-rays travel in straight lines.
ii. X-rays cannot be deflected by electric field or magnetic field.
iii. X-rays have a high penetrating power.
iv. Photographic film is blackened by X-rays.
v. Fluorescent materials glow when X-rays are directed at them.

vi. Photoelectric emission can be produced by X-rays.
vii. Ionization of a gas results when an X-ray beam is passed through it.

Continuous Spectrum

When the accelerated electrons (cathode rays) strike the metal target, they collide with electrons in the target. In such a collision part of the momentum of the incident electron is transferred to the atom of the target material, thereby losing some of its kinetic energy, ΔK. This interaction gives rise to heating of the target. The projectile electron may avoid the orbital electrons of the target element but may come sufficiently close to the nucleus of the atom and come under its influence. The loss in kinetic energy reappears as an x-ray photon. During deceleration, the electron radiates an X-ray photon of energy. $h\nu = \Delta K = K_f - K_i$, The resulting spectrum is continuous but with a sharp cut-off wavelength. The minimum wavelength corresponds to an incident electron losing all of its energy in a single collision and radiating it away as a single photon. If K is the kinetic energy of the incident electron, then

$$K = h\nu = \frac{hc}{\lambda_{min}}$$

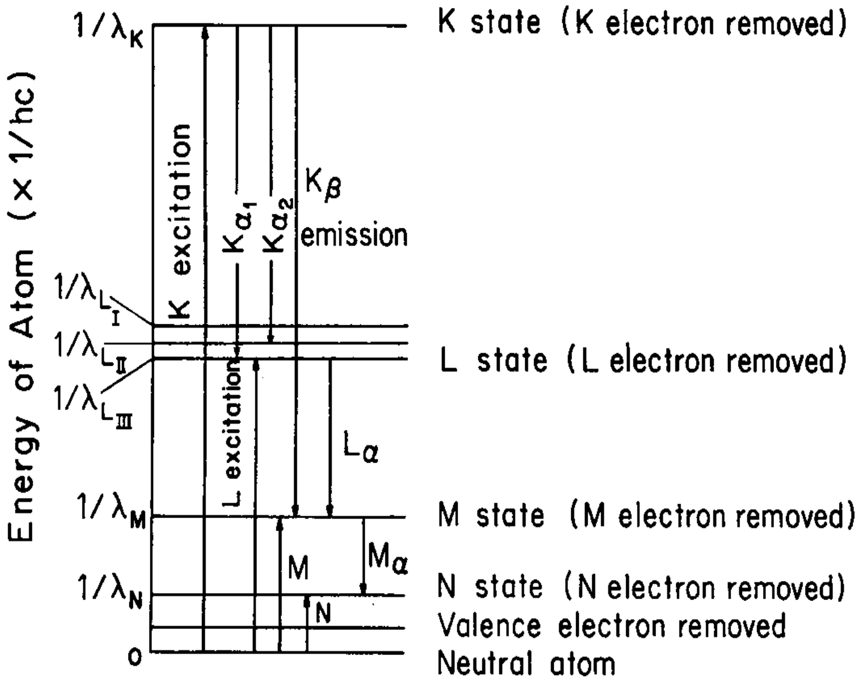
Because of the large accelerating voltage, the incident electrons can

(i) Excite electrons in the atoms of the target.

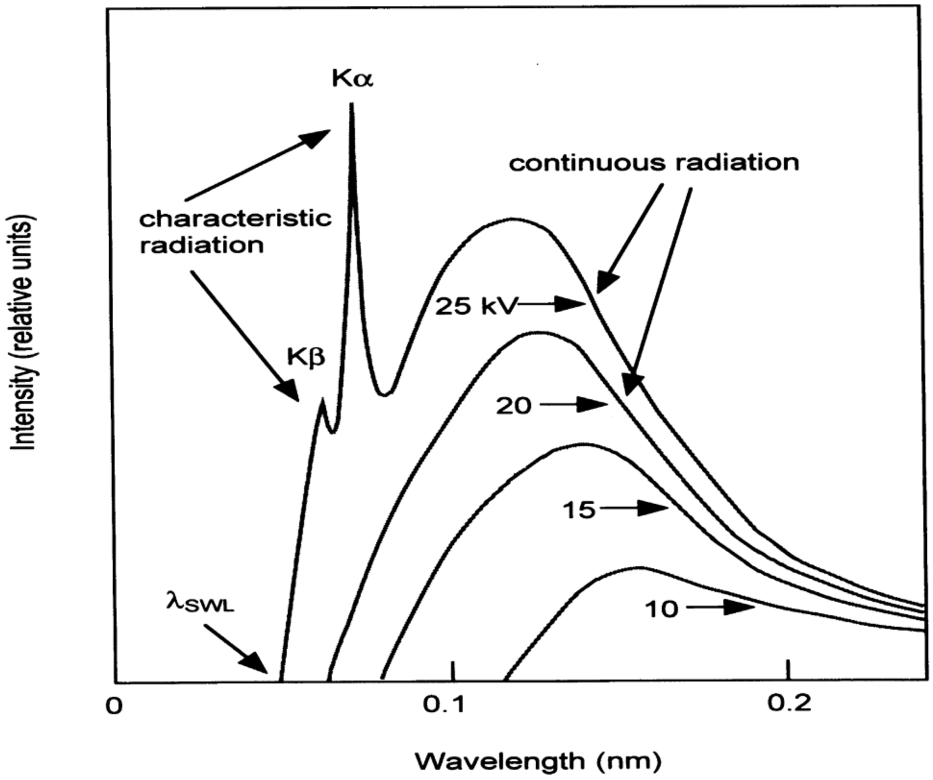
(ii) Eject tightly bound electrons from the cores of the atoms.

Characteristic X-Ray Spectrum

Excitation of electrons will give rise to emission of photons in the optical region of the electromagnetic spectrum. However when core electrons are ejected, the subsequent filling of vacant states gives rise to emitted radiation in the x-ray region of the electromagnetic spectrum. The core electrons could be from the K-, L- or M- shell. If K-shell (n=1) electrons are removed, electrons from higher energy states falling into the vacant K-shell states, produce a series of lines denoted as Kα, Kβ ,... as shown in the Figure below. Transitions to the L shell result in the L series and those to the M shell give rise to the M series, and so on. Since orbital electrons have definite energy levels, the emitted X-ray photons also have well defined energies. The emission spectrum therefore has sharp lines characteristic of the target element.



X–Ray Transitions

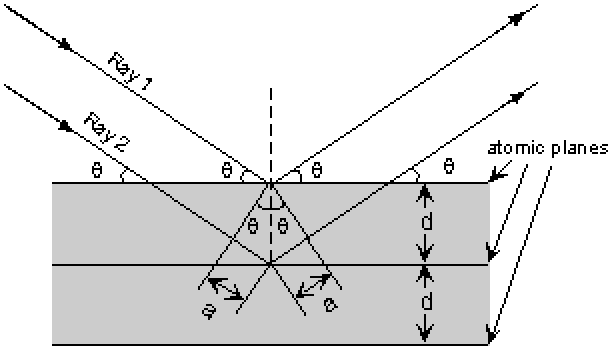


The graph shows the following features:

A continuous background of X-radiation in which the intensity varies smoothly with wavelength. The background intensity reaches a maximum value as the wavelength increases, and then the intensity falls at greater wavelengths. The minimum wavelength depends on the tube voltage. The higher the voltage the smaller the value of the minimum wavelength. The sharp peaks of the intensity distribution occur at wavelengths that is independent of the change in the tube voltage.

X-Ray Diffraction

A plane of atoms in a crystal, also called a Bragg plane, reflects X-ray radiation in exactly the same manner that light is reflected from a plane mirror. **Reflection from successive planes can interfere constructively if the path difference between two rays is equal to an integral number of wavelengths.** This statement is called Bragg’s law.



Thus, the condition for constructive interference to occur is

$n\lambda = 2a$

but, from trigonometry, we can figure out what the distance 2a is in terms of the spacing, d, between the atomic planes.

$a = d \sin \theta$

or $2a = 2 d \sin \theta$

thus, **$n\lambda = 2d \sin \theta$**

This is known as **Bragg's Law** for X-ray diffraction.

What it says is that if we know the wavelength ,λ , of the X-rays going in to the crystal, and we can measure the angle θ of the diffracted X-rays coming out of the crystal, then we know the spacing (referred to as ***d-spacing***) between the atomic planes.

$d = n\lambda /2 \sin \theta$

Again it is important to point out that this diffraction will only occur if the rays are in phase when they emerge, and this will only occur at the appropriate value of n (1, 2, 3, etc.) and θ.

In theory, then we could re-orient the crystal so that another atomic plane is exposed and measure the d-spacing between all atomic planes in the crystal, eventually leading us to determine the crystal structure and the size of the unit cell.

Moseley's Experiment

The high intensity penetrating radiation emitted by X-ray tubes, characteristic of the metal from which the target anode is made, was first discovered by Barkla. Changing the metal or element from which the target anode in the X-ray tube is made alters the wavelengths at which the high intensity peaks occur. The most penetrating series in an element's characteristic X-ray spectrum is called the K series; the second is called the L series; the third the M series and so on. Moseley carried out a systematic examination of the characteristic radiation of as many elements as possible. Moseley discovered a simple empirical relationship between the frequencies, (ν) of the lines in each series and the ordinal number, Z, of the element's position in the periodic table (starting from hydrogen):

$$\sqrt{\nu} = b(Z-a)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

ν = Frequency of characteristic radiation

b = Constant which is different for different series

a = Constant known as screening constant and is different for different series

$$b = \frac{m\,e^4}{8\epsilon_0^2 h^3}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right),$$ n_1 and n_2 are principal quantum numbers

For K_x line, b was found to be equal to (¾)R, where R is Rydberg constant and 'a' was found to be practically a = 1, hence for K_α line

$$\nu_{K_\alpha} = \frac{3}{4}R(Z-1)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Equation (1) is known as Moseley law or Moseley equation.

The exact form of Moseley law is

$$\frac{1}{\lambda} = R(Z-\sigma)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where σ is a correction factor and n_1 and n_2 are the principal quantum numbers of the energy levels between which the transition occur.

The square root $\sqrt{\nu}$ of the frequency of an element's K line as a function of the ordinal number, N, of its position in the periodic table.

Moseley formed the opinion that some physical attribute of the atom must increase by (a) regular fixed amount, from one element to the next, rising through the periodic table. He postulated that this could only be the atom's nuclear charge.
According to this hypothesis, the number N, that is the element's ordinal position in the periodic table, is equal to the number of natural units of positive electricity carried by the nuclei of the element, i.e., N=Z . The number Z is now called the *atomic number* of the element; it is equal to the number of protons in the element's nuclei. Prior to Moseley's investigation, the elements were arranged in the periodic table in the ascending order of their atomic weights and on the basis of their chemical properties. As a result of Moseley's researches, which provided the first direct means of determining an element's atomic number, inaccuracies in the periodic table were discovered and corrected.

Example 1
Find the minimum wavelength of X-rays produced by an X-ray tube operated at 1000 kV. If $h = 6.63 \times 10^{-34}\, joules - sec,\ e = 1.6 \times 10^{-19}\, C\ and\ c = 3 \times 10^{8}\, m/sec$

$$\lambda_{min} = \frac{hc}{eV} = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{(1.6 \times 10^{-19})(1000 \times 10^{3})}$$

$$= 0.01237 \times 10^{-10}\, m\ = 0.01237\text{Å}$$

Example 2
If the potential difference applied across an X-ray tube is 5kV and the current flowing through it is 2 mA, calculate

i. The number of electrons striking the target per second
ii. The speed at which they strike

Solution

i. $I = ne$, where n is the number of electrons striking the anode per second.

$$n = \frac{I}{e} = \frac{2 \times 10^{-3}}{1.6 \times 10^{-19}} = 1.25 \times 10^{16}\ electrons$$

ii. If v is the velocity of striking electrons

$$\frac{1}{2}mv^2 = eV$$

$$\sqrt{\left\{\frac{2 \times (1.6 \times 10^{-19}) \times (50 \times 10^3)}{9.1 \times 10^{-31}}\right\}} = 4.2 \times 10^7 ms^{-1}$$

Example 3
The spacing between the principal planes of a NaCl crystal is 2.82 Å. It is found that the first order Bragg reflection occur at an angle of 10˚. Calculate the wavelength of X-rays.

Solution:
According to Bragg"s law

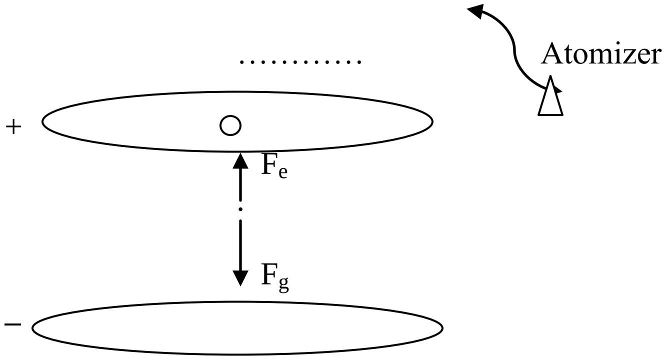
$$2d \sin\theta = n\lambda$$

$$\lambda = \frac{2d \sin\theta}{n} = \frac{2 \times (2.82 \times 10^{-10}) \times \sin 10^{\circ}}{1}$$

$$\lambda = 0.98 \times 10^{-10} m = 0.98\text{Å}$$

Millikan's Oil-Drop Experiment

In 1909 Robert A. Millikan came up with an experiment to measure the charge on an electron, called the Oil Drop Experiment.
The apparatus was actually quite simple. There were two parallel plates set at a specific distance apart with a known voltage between them. That way we know the electric field strength. The top plate is positive, and the bottom plate is negative. Millikan drilled a very small hole in the center of the top plate. He then used an atomizer to spray very fine drops of mineral oil over the top plate. An atomizer is like those fancy perfume bottles you see that have a ball you squeeze to make the perfume spray out. Friction between the nozzle of the atomizer and the mineral oil droplets caused some of the drops to gain a small charge (charging by friction).



Illustrating the Milikan"s Oil Drop Experiment

Just by chance, some of the oil drops might fall down the hole in the top plate. If they have a positive charge, we expect them to go accelerating down to the negative plate and crash into it. If they have a negative charge, something different might happen. If the force due to gravity (Fg) pulling the drop down is exactly balanced by the electric force (Fe) pushing it up, the drop should float between the two plates. Since the force due to the electric field and the force due to gravity are balanced, it is possible to derive an equation to calculate the charge on the droplet.

$$F_e = F_g, \qquad qE = mg, \qquad q\frac{V}{d} = mg, \qquad q = \frac{mgd}{V}$$

q is the charge, m is mass of oil droplet, g is acceleration due to gravity, d is distance between plates and V is the voltage.

After thousands of trials, Millikan had enough successful trials to show that all of the charges he calculated were multiples of one number, $1.6 \times 10^{-19} C$.

Example

An oil drop in a Millikan apparatus is determined to have a mass of 3.3e-15 kg. It is observed to float between two parallel plates separated by a distance of 0.95cm with 340V of potential difference between them. Determine how many excess (extra) electrons are on the drop.

$$q = \frac{mgd}{V} = \frac{3.3 \ \times 10^{-15} \times 9.8 \times 0.0095}{340}$$

$$q = 9.0 \times 10^{-19} C$$

.

$$\text{Number of charges} = \frac{9.0 \times 10^{-19} C}{1.6 \times 10^{-19} electrons} = 5.65337316 \ electrons$$