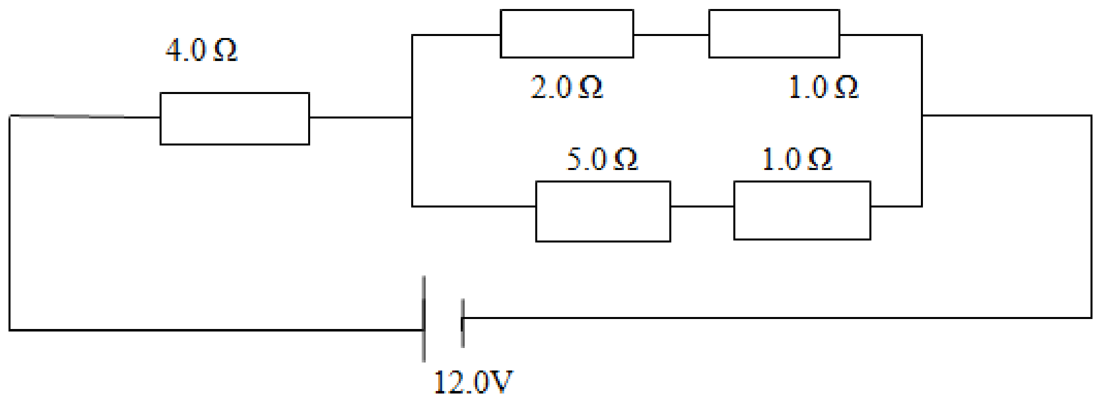
PHY 152: Electricity and Magnetism I

Please take note of typo error in some questions and answers provided under “Ohm’s Laws, Kirchhoff’s Laws and Electrical Energy”.

The affected questions together with the answers are given below.



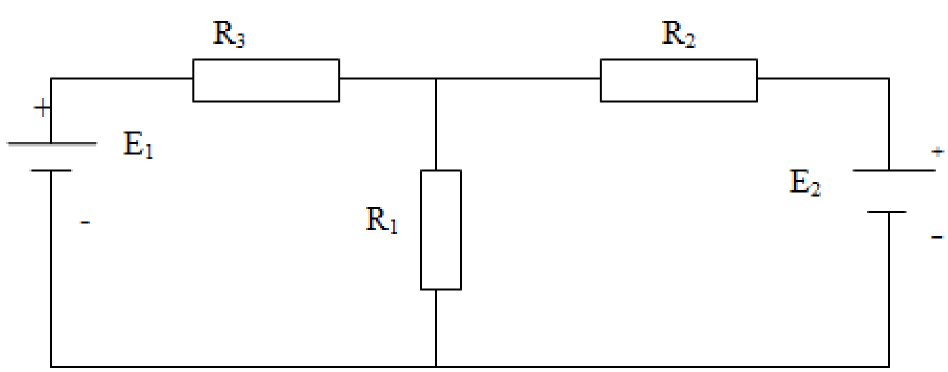
Determine the power dissipated in the 2.0 Ω resistor in the circuit above (Resistive dissipation)

Ans: power dissipated = 3.56 W

A copper wire and aluminum wire have the same length. Obtain the ratio of diameter of aluminum to that of copper if the resistance of copper is twice that of aluminum and the resistivity of copper $\rho_c = 1.72 \times 10^{-8}$ Ωm and that of aluminum $\rho_a = 2.82 \times 10^{-8}$ Ωm. Ans: 9:5

In the diagram below, $E_1 = 3.0V$, $E_2 = 1.00V$, $R_1 = 5.0\ \Omega$, $R_2 = 2.0\ \Omega$, $R_3 = 4.0\ \Omega$ and both batteries are ideal. What is the rate at which energy is dissipated in (a) R_1 (b) R_2 (c) R_3

Ans: $P_1 = 0.35$ W $P_2 = 0.05$ W & $P_3 = 0.71$ W



PHY152: ELECTRICITY AND MAGNETISM

SUB-TOPIC

Coulombs Laws, Gauss Laws and Electric Potentials

Coulombs Law:

- The magnitude of the Electric Force exerted by one point charge on the other is directly proportional the product of the charges and inversely proportional to the square of the distance r btw them

$$F = k\frac{Q_1 Q_2}{r^2}$$

if $F = +ve \rightarrow$ repulsive Force

$F = -ve$ — attractive force

$k = 8.99\times10^{9}\ Nm^2/C^2 = \frac{1}{4\pi\varepsilon_0}$

ε_0 permittivity of free spa

$\varepsilon_0 = 8.85\times10^{-12}\ C^2/Nm^2$ //

Exercise:

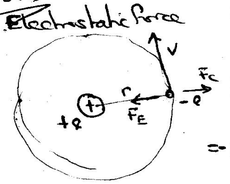
1. In the Bohr model of the Hydrogen atom, the electron is in orbit about the proton at a radius of 5.29×10^{-11} m. ($m_e = 9.11\times10^{-31}$ kg)

a. Determine the force on the electron.

b. Find the speed of the electron

Soln

a. Electrostatic force   Hydrogen = 1 proton
 = 1 electron



$$F_E = k\cdot\frac{q_1 q_2}{r^2}$$

$$= -8.99\times10^{9}\times\frac{(1.6\times10^{-19})^2}{(5.29\times10^{-11})^2}$$

$$= -8.22\times10^{-8}\ N //$$

Gravitational force $\simeq 0$ since both the mass of proton and the mass of electrons are very small

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b. Centripetal force

$$F_c = \frac{mv^2}{r}$$

since the radius is constant,

$$F_E = F_c$$

$$\Rightarrow F_E = \frac{mv^2}{r}$$

$$\Rightarrow v^2 = \frac{rF_E}{m}$$

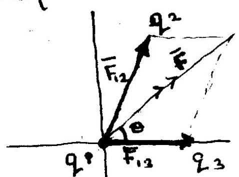
$$\Rightarrow v = \sqrt{\frac{(5.29\times10^{-11})(8.22\times10^{-8})}{9.11\times10^{-31}}}$$

$$= 2.18\times10^{6}\ m/s //$$

Vectors Quantities

They have both magnitude and direction

eg



Resultant force on q_1 from q_2 and q_3

$$\bar{F} = \bar{F}_{12} + \bar{F}_{13}$$

x component, y component

$\bar{F}$ can be resolved into its x and y components (x component, y component)

as $\bar{F} = \bar{F}_x + \bar{F}_y$

Hence, we can draw a table such as

Hence, we can draw a table such as

Force	x component	y component
$\bar{F}_{12}$	$+\lvert F_{12}\rvert\cos\theta_{12} = F_{12x}$	$+\lvert F_{12}\rvert\sin\theta_{12} = F_{12y}$
$\bar{F}_{13}$	$+\lvert F_{13}\rvert\cos\theta_{13} = F_{13x}$	$+\lvert F_{13}\rvert\sin\theta_{13} = F_{13y}$
$\bar{F}$	$\bar{F}_x$	$\bar{F}_y$

ii The magnitude F and the angle of the net force are

$$F = \sqrt{F_x^2 + F_y^2}$$

and

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

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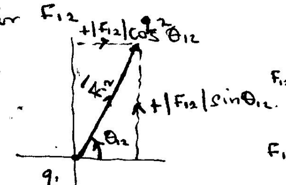
i For F_{12}



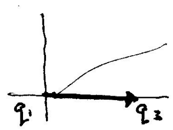
x component

$F_{12x} = |F_{12}|\cos\theta_{12}$

y component

$F_{12y} = +|F_{12}|\sin\theta_{12}$

ii For F_{13}



x component

$\theta_{13} = 0$ $\quad F_{13x} = +|F_{13}|\cos\theta_{13}$

$= +|F_{13}|\cos 0$

$= +|F_{13}|$

y component

$F_{13y} = +|F_{13}|\sin\theta_{13}$

$= +|F_{13}|\sin 0$

$= 0$

Summary:

Net force on any one charge

~~$\bar{F} = \Sigma \bar{F}_x + \Sigma \bar{F}_y$~~

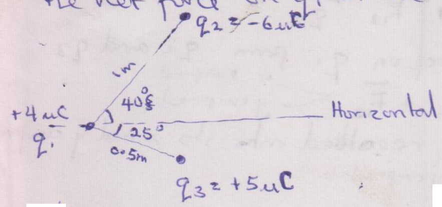
$$\bar{F} = \bar{F}_x + \bar{F}_y$$

$\bar{F}_x = \Sigma$ Force component in the x direction

$\bar{F}_y = \Sigma$ Force component in the y direction.

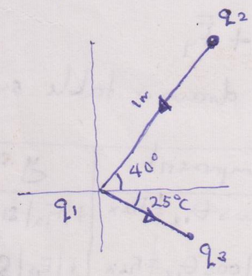
Example

2. Find the magnitude and direction of the net force on q_1 in the diagram below



Soln

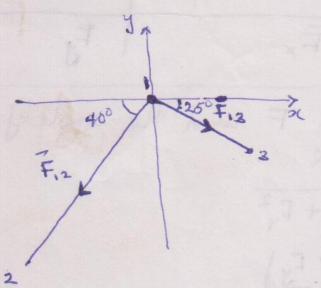
Step 1



i Fix q_1 on the zero of the vector grid

ii - draw the charges on the vector grid indicating the direction of force

step 2

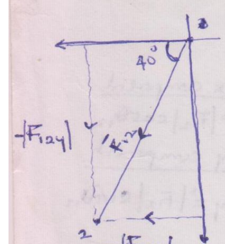


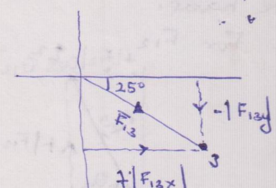
i - Draw all the vectors such that they all flow out from the zero of the force grid

ii - Measure all angles relative to the x axis

Step 3

i Break down each force along its x and y directions on the vector grid





ii note the direction of the forces, -ve or +ve of the xy grid

Step 4: Write the equation for each of these force

$F_{12x} = -|F_{12}| \cos 40^\circ$ $\quad$ $F_{13x} = +|F_{13}| \cos 25^\circ$

$F_{12y} = -|F_{12}| \sin 40^\circ$ $\quad$ $F_{13y} = -|F_{13}| \sin 25^\circ$

Step 5
To obtain the resultant Force F.

$\bar{F} = \bar{F}_x + \bar{F}_y$ $\quad$ and $\bar{F}_y = F_{12y} + F_{13y}$.

$\bar{F}_x = F_{12x} + F_{13x}$

$F_x = -|F_{12}|\cos 40^\circ + |F_{13}|\cos 25^\circ$

$F_y = -|F_{12}| \sin 40^\circ - |F_{13}| \sin 25^\circ$

Step 5 : Compute the magnitude $|F_{12}|$ and $|F_{13}|$

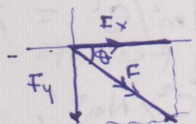
$$|F_{12}| = k\frac{q_1 q_2}{r_{12}^2} = 8.99\times10^9 \frac{4\times10^{-6}\times 6\times10^{-6}}{(1.0)^2} = 0.216\text{N}$$

$$|F_{13}| = k\frac{q_1 q_3}{r_{13}^2} = 8.99\times10^9 \frac{4\times10^{-6}\times 5\times10^{-6}}{(0.5)^2} = 0.719\text{N}$$

$\Rightarrow F_x = -0.216\cos 40 + 0.719 \cos 25$
$= -0.165 + 0.652$
$= 0.49\text{ N}$

$F_y = -0.216 \sin 40 - 0.719 \sin 25$
$= -0.139 - 0.304$
$= -0.44\text{ N}$

Step 6



Magnitude of Force $F = \sqrt{F_x^2 + F_y^2}$
$= \sqrt{0.49^2 + 0.44^2}$
$= 0.66\text{ N}$

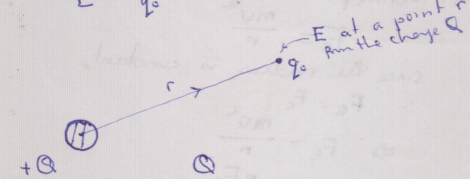
Direction θ.

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$
$= \tan^{-1}\left(\frac{-0.44}{0.49}\right)$
$= -41.9^\circ$

Electric Field E

The electric field of a charge Q at a point is the force it exerts per unit charge when a small test charge is placed at that point.

i.e. $\bar{E} = \frac{F}{q_0}$ $\quad$ unit (N/C



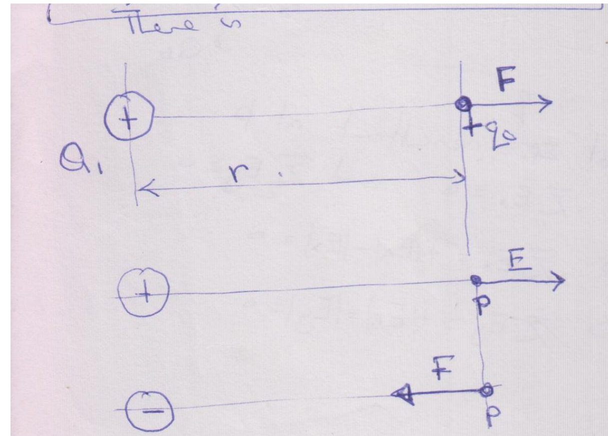
$\bar{E} = k\frac{Q}{r^2}$

E = +ve for a +ve charge E radiates outward from Q
= -ve for a -ve charge E radiates inward to Q.

Electric field:

The electric field E that exists at a point is the electrostatic force F experienced by a small test charge q_0 placed at that point divided by the charge itself:

$$E = \frac{F}{q_0} = k\frac{Q}{r^2}$$ //.



Electric Field Produced by Point Charges

Exercise:

1. There is an isolated point charge of $Q = +15\mu C$. Using a test charge, $q_0 = +0.8\mu C$, (a) determine the electric field at a point P, which is 20 cm away.

b What happens if the test charge is $0.1\mu C$.

Soln.

a. $$E = \frac{F}{q_0} = k\frac{q_0 Q}{q_0 r^2}$$

$$= k \times \frac{(0.8\times10^{-6})\times(15\times10^{-6})}{(0.2)^2\cdot(0.8\times10^{-6})}$$

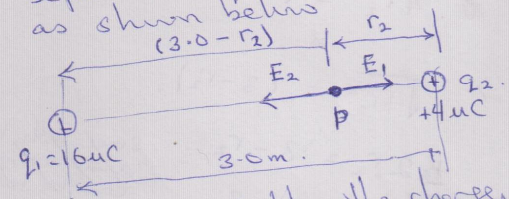
$$= 3.4\times10^{6}\ N/C$$ //.

b. $E = \frac{F}{q_0}$

$= k \frac{\cancel{q_0} Q}{\cancel{q_0} r^2}$

$= \frac{(15\times10^{-6})}{(0.02)^2} = 3.4\times10^{6}$ N/C

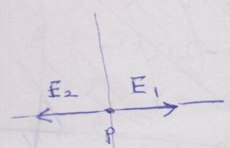
2. Two positive point charges $q_1 = +16\mu C$ and $q_2 = +4\mu C$, are separated by a distance of 3.0 m. as shown below



Find the spot b/w the charges where the net electric field is zero.

Soln

Draw the vector diagram with P at the centre of your grid



The net electric field is zero when

$\sum E_x = 0$, $\sum E_y = 0$ and $\sum E_z = 0$

Hence $\sum E_x = E_1 - E_2 = 0$

$\Rightarrow E_1 = E_2$

$E_1 = k \frac{Q_1}{(3.0 - r_2)^2}$

$E_2 = k \frac{Q_2}{r_2^2}$

Since $E_1 = E_2$

$\cancel{k} \frac{Q_1}{(3 - r_2)^2} = \cancel{k} \frac{Q_2}{r_2^2}$

$\frac{16\times10^{-6}}{(3 - r_2)^2} = \frac{4\times10^{-6}}{r_2^2}$

Re-arranging gives.

$$\frac{r_2^2}{(3-r_2)^2} = \frac{4\times10^{-16}}{16\times10^{-16}}$$

$$\Rightarrow \frac{r_2^2}{(3-r_2)^2} = \frac{1}{4}$$

$$4r_2^2 = (3-r_2)^2$$

$$\Rightarrow \sqrt{4r_2^2} = (3-r_2)$$

$$2r_2 = 3-r_2$$

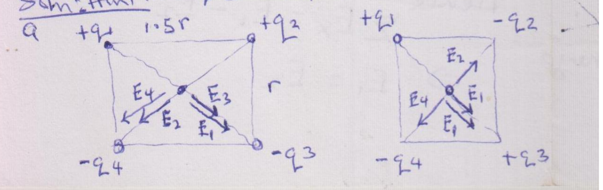
$$\Rightarrow 2r_2 + r_2 = 3$$

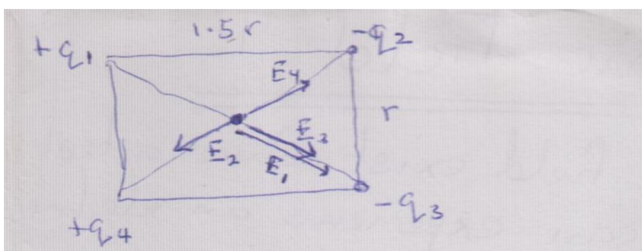
$$\Rightarrow 3r_2 = 3$$

$$\Rightarrow r_2 = \frac{3}{3} = 1\,m$$

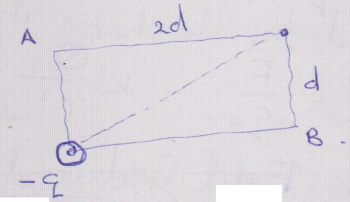
3. Find the electric field strength at the centre of each of the figures of point charges where All q's are equal

Soln: Hint:-

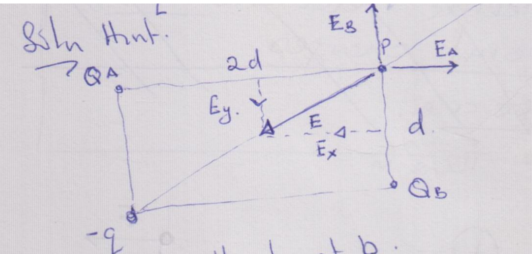




4. A negative charge -q is fixed to one corner of a rectangle as in the drawing below. What +ve charge must be fixed to corner A and what +ve charge must be fixed to corner B, so that the electric field at the remaining corner is zero. Express your answers in terms of q.



Soln Hint.



for zero resultant at p.

$\sum E_x = 0$ and $\sum E_y = 0$

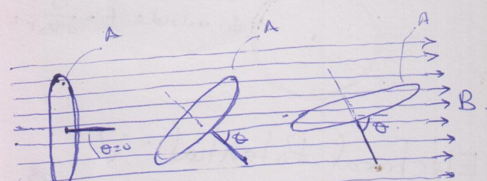
$\Rightarrow \sum E_x = +|E_A| - |E_x| = 0$

$\Rightarrow \sum E_y = +|E_B| - |E_y| = 0$

Gauss's Law

The Coulombs laws may be expressed in a form known as Gauss's Law. However, the number of problems that can be solved using the Gauss formulation is quite limited. because the charge distribution must have a special symetric property.

Consider a plane surface area cutting into some magnetic field lines as shown below

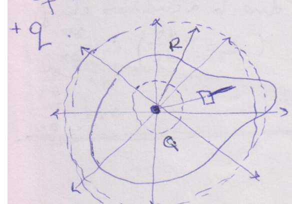


θ = Angle btw the normal to the plane and the magnetic field line (flux).

The (number of flux lines){flux} passing through each surface area is given by

$\phi = EA\cos\theta$ // //

Consider an arbitrary closed surface of area A enclosing a charge +q



$\phi = \oint (E\cdot\hat{n})\,ds$ where ds = elemental surface area

$\phi = \oint E \cdot \oint ds = E \cdot 4\pi R^2$

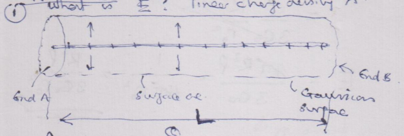
$= \frac{Q}{4\pi\varepsilon_0 R^2} \cdot 4\pi R^2 = \frac{Q}{\varepsilon_0}$

Hence

$\oint (E\cdot\hat{n})\,ds = \frac{Q}{\varepsilon_0}$

Application:

① ~~What is~~ E? linear charge density λ.



$$\oint (E \cdot \hat{n})\, ds = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \int E \cdot n\, ds_A + \int E \cdot n\, ds_C + \int E \cdot n\, ds_B = \frac{Q}{\varepsilon_0}$$

$$0 + E \cdot (2\pi R L) + 0 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{Q}{2\pi\varepsilon_0 R L} \quad //$$

If linear charge density $= \lambda$.

$$\Rightarrow \lambda = \frac{Q}{L}$$

$$\Rightarrow Q = \lambda L$$

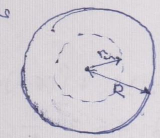
$$\Rightarrow E = \frac{\lambda \cancel{L}}{2\pi\varepsilon_0 R \cancel{L}} = \frac{\lambda}{2\pi\varepsilon_0 R} \quad //$$

② Solid Sphere of radius R uniformly charged to a volume density ρ.

ⓐ What is E inside ⓑ What is E outside

Soln

ⓐ Inside



$$\rho = \frac{Q}{V_{in}} = \frac{Q}{4/3\,\pi r^3} = \frac{3Q}{4\pi r^3}$$

$$\oint (E \cdot \hat{n})\, ds = \frac{Q_{in}}{\varepsilon_0}$$

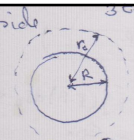
$$\Rightarrow Q = \frac{4\pi r_{in}^3 \rho}{3}$$

$$\Rightarrow E \cdot (4\pi r_{in}^2) = \frac{\frac{4}{3}\pi r^3 \rho}{\varepsilon_0} = \frac{4\pi r^3 \rho}{3\epsilon_0}$$

$$\Rightarrow E = \frac{\cancel{4\pi} r_{in}^{\cancel{3}} \rho}{3\epsilon_0} \cdot \frac{1}{\cancel{4\pi} \cancel{r_{in}^2}}$$

$$= \frac{\rho r}{3\epsilon_0} \quad //$$

b. Outside



$$\oint (E \cdot \hat{n})\, ds = \frac{Q_{in}}{\varepsilon_0}$$

$$\therefore E \cdot (4\pi r_o^2) = \frac{\frac{4}{3}\pi R^3 \rho}{\epsilon_0} \qquad r_o > R.$$

$$\cancel{E = \frac{4\pi R^3 \rho}{3\epsilon_0} \cdot \frac{1}{4\pi R_o^2}}$$

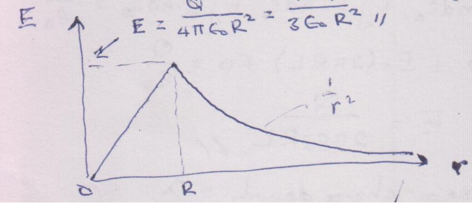
$$\cancel{= \frac{\rho}{3\epsilon_0} \frac{R^3}{r_o^2}}$$

$$E = \frac{\cancel{4\pi} R^3 \rho}{3\epsilon_0} \cdot \frac{1}{\cancel{4\pi} r_o^2} = \frac{R^3 \rho}{3\epsilon_0 r_o^2} \quad //$$

in terms of Q.

$$E = \frac{Q}{\epsilon_0} \cdot \frac{1}{4\pi r_o^2} = \frac{Q}{4\pi\epsilon_0 r^2} \quad //$$

$$E = \frac{Q}{4\pi\epsilon_0 R^2} = \frac{R^3 \rho}{3\epsilon_0 R^2} \quad //$$



(3) A very long cylindrical uniformly charged cylinder with charge density ρ, what is E inside and outside?

Soln

(a) Inside $(r < R)$

$$\rho = \frac{Q_{enclosed}}{vol} = \frac{Q_{enclosed}}{(\pi r^2 L)} \Rightarrow Q = \pi r^2 L \rho$$

By Gauss law

$$\oint (E \cdot \hat{n}) \cdot ds = \frac{Q}{\epsilon_0}$$

$$\Rightarrow \int (E \cdot \hat{n}) ds_A + \int (E \cdot \hat{n}) ds_s + \int (E \cdot \hat{n}) ds_B = \frac{Q}{\epsilon_0}$$

$$0 + E \int ds + 0 = \frac{Q}{\epsilon_0}$$

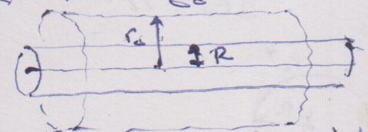
$$E \cdot (2\pi r L) = \frac{\pi r^2 L \rho}{\epsilon_0}$$

$$E = \frac{\cancel{\pi} r^{\cancel{2}} \cancel{L} \rho}{2 \cancel{\pi} \cancel{r} \cancel{L} \epsilon_0} = \frac{\rho r}{2 \epsilon_0} ///$$

(b) Outside

By Gauss law

$$\oint (E \cdot \hat{n}) ds = \frac{Q_{in}}{\epsilon_0}$$



$$\int (E \cdot \hat{n}) ds_A + \int (E \cdot \hat{n}) ds_s + \int (E \cdot \hat{n}) ds_B = \frac{Q_{in}}{\epsilon_0}$$

$$0 + \hat{E} \cdot \int ds + 0 = \frac{Q_{in}}{\epsilon_0}$$

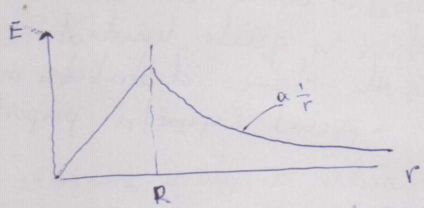
$$E \cdot (2\pi r L) = \frac{Q_{in}}{\epsilon_0}.$$

$$E = \frac{1}{2\pi r L} \cdot \frac{Q_{in}}{\epsilon_0}.$$

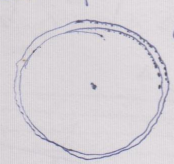
$$Q = \pi R^2 L \rho$$

$$\Rightarrow E = \frac{1}{2 \cancel{\pi} r \cancel{L}} \cdot \frac{\cancel{\pi} R^2 \cancel{L} \rho}{\epsilon_0}$$

$$= \frac{\rho R^2}{2 r \epsilon_0} = \frac{\rho R^2}{2 \epsilon_0} \cdot \frac{1}{r}.$$



(4) Hollow spherical charged surface



(a) inside $E = 0$

(b) outside $E = \frac{Q}{4\pi\epsilon_0 r^2}$

Electrical Potential: V