PHY 152 (S. Olatunji)

Content: Magnetism, Hysteresis, Power, AC circuit (Vector)

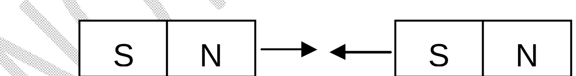
Magnetism

Magnetism is expressed inform of force of attraction or repulsion between various substances, especially those made of iron and certain other metals; ultimately it is due to the motion of electric charges. Unlike electric forces, which or not, magnetic forces act only on moving charges.

Magnetic forces arise in two stages. First, a moving charge or a collection of moving charges (i.e. an electric current) produces a magnetic field. Next, a second current or moving charge responds to this Magnetic field, and so experiences a magnetic force.

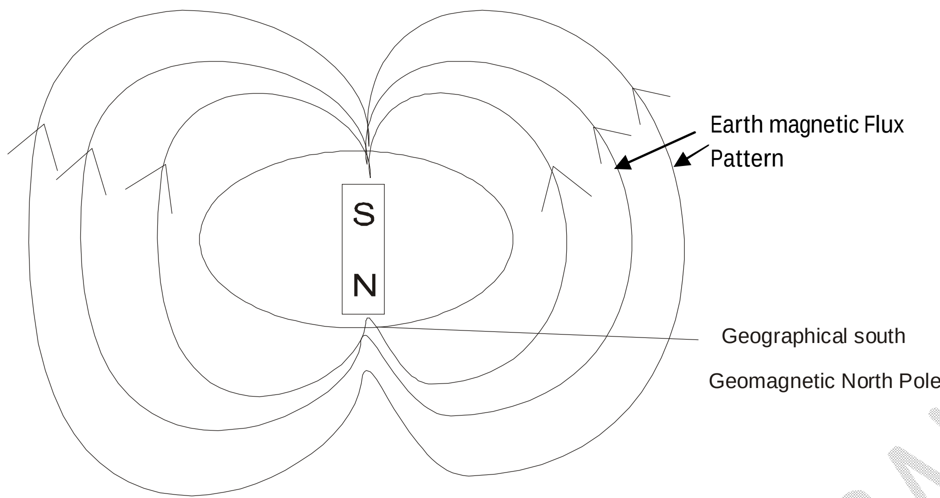
A permanent magnet may have two or more poles, although it must have at least one North Pole and one South Pole. The earth itself behaves like a magnet. Its north geographic pole is close to a magnetic south pole, which is why the north pole of a compass needle points to the geographical north (magnetic South Pole) because magnetic field lines (i.e. flux) exit North Pole and enter South poles.

Magnetic poles of the same type (North or South) repels each other while unlike poles attract each other (Benjamin's rule)



Unlike poles attract

Like poles repel



The sketch above is the Earth's Magnetic field. The lines, called magnetic field lines, show the direction that a compass would point at each location. The direction of the field at any point can be defined as the direction of the force that the field would exert on a magnetic north pole.

Magnetic Field $(\vec{B})$

A Magnetic field $(\vec{B})$ exists in an otherwise empty region of space if a charge moving through that region can experience a force due to its effect on a compass needle the magnetic field is a vector field (i.e., a vector quantity) associated with each point in space, hence, the use of the symbol B. A moving charge or a current creates a magnetic field in the surrounding space and the magnetic field exerts a force F on any other moving charge or current that is present in the field.

Magnetic Forces on moving Charges

There are four characteristics of the magnetic force on a moving charge. First, its magnitude is proportional to the magnitude of the charge. If a 1μC charge and a 2μC charge move through a given magnetic field with the same velocity, experiments show that the force on the 2μC charge is twice as great as the force on the 1μC charge.

Second, the magnitude of the force is also proportional to the magnitude, or "Strength", of the field; if we double the magnitude of the field. (e.g. using two identical bar magnets instead of one) without changing the charge or its velocity, the force doubles.

A third characteristic is that the magnetic force depends on the particles velocity. A charged particle at rest experiences no magnetic force.

Fourth, we find, by experiment, that the magnetic force $\vec{F}$ does not have the same direction as the magnetic field $\vec{B}$ but instead is always perpendicular to both $\vec{B}$ and the velocity $\vec{V}$. The Magnitude $\vec{F}$ of the force is found to be proportional to the component of $\vec{V}$ perpendicular to the field; when that component is zero (i.e, when $\vec{V}$ and $\vec{B}$ are parallel or antiparallel), the force is zero.

The Magnitude of the force (F) on a charge moving on a magnetic field depends upon the product of four factors:

1. q, the charge (in Coulombs)
2. V, the magnitude of the velocity of the charge (in m/s)
3. B, the strength of the magnetic field.
4. Sinϴ, where ϴ is the angle between the field lines and the velocity V.

$$F = q\, V\, B \sin\theta.$$

Where F is in Newton, q is in Coulomb, V is in m/s, and B is in Tesla (T).

$1T = 1Wb/m^2$, $1G = 10^{-4}T$, where G is Gauss.

Since current is simply a stream of positive charges, a current experiences a force due to a magnetic field. The direction of the force is found by the Fleming's right-hand rule.

The magnitude ΔF_m of the force on a small length ΔL of wire carring current I is given by

$$\Delta F_m = I\,(\Delta L)\, B \sin\theta.$$

Where, ϴ is the angle between the direction of the current I and the direction of the field. For a single wire of length L in a uniform magnetic field, this becomes.

$$F_m = I\, L\, B \sin\theta$$

The force is zero if the wire is in line with the field lines and the force is maximum if the field lines are perpendicular to the wire.

Force and Torque of a Loop

The net force on a current loop in a uniform magnetic field is zero. However, the net torque is not in general equal to zero.

The magnetic moment of a flat current – carrying loop (current =I, area = A) is IA. The magnetic moment is a vector quality that points along the field line perpendicular to the plane of the loop. In terms of the magnetic moment, the torque on a flat coil with N loop in a magnetic field B is

μ= IA

τ= (IA) B sinΘ

τ = μB sinΘ

Where, μ= magnetic moment

τ = Torque

Θ is the angle between the field and the magnetic moment vector.

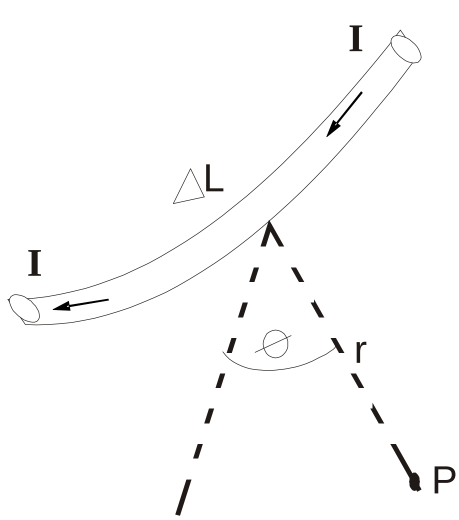
For a coil of N loop, each carrying I, in an external magnetic field B. the torque τ is given as

τ =NIAB sinΘ

Magnetic Field of a Current Element

The current element of lengthΔL contributes$\overrightarrow{\Delta B}$ to the field at a point is given by the Biot - savart law *viz:*

$$\Delta B = \frac{\mu_0 I \Delta L \sin\theta}{4\pi r^2}$$



Where r and Θ are radius and angle to the point as show above. The direction of ΔB is $\overrightarrow{\text{per}}$pendicular to the place determined by ΔL and r. when r is in line with ΔL, then $\Theta = 0$ and thus $\Delta B = 0$; which means that the field due to a straight wire at a point on the line of the wire is zero.

Magnetic Flux

The magnetic flux $\emptyset_m$ through an area A is defined to be the product of $B_\perp$ and A, where $B_\perp$ is the component of $\vec{B}$ perpendicular to the surface of area A

$$\emptyset_m = B_\perp A = BA\cos\Theta$$

$$B_\perp = B\cos\Theta.$$

Where Θ is the angle between the direction of $\vec{B}$ and a line perpendicular to the surface.

Magnetic flux is a Vector quantity. If $\vec{B}$ is uniform over a plane surface with total area A, then B and $\Theta_\perp$ are the same at all points on the surface, and

$$B_\perp = B\cos\Theta.$$

If $\vec{B}$ happens to be perpendicular to the surface, then $\cos\Theta = 1$ and $B = B_\perp$ i.e. $\emptyset_m = BA$. Its S.I unit is weber or Tesla metre square ($1Wb = 1T.m^2$).

The total magnetic flux through a closed surface is always zero

$$\text{i.e} \quad \int \vec{B} \cdot \vec{dA} = 0$$

This equation is called Gauss's law of magnetism.

Induced EMF

An induced emf exists in a loop of wire whenever there is a change in the magnetic flux through the area surrounded by the loop. The induced emf exists only during the time that the flux through the area is changing.

Faraday's law of induction states that the induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop.

$$\varepsilon = \frac{-d\,\emptyset_m}{dt}$$

The emf ε is measured in volts if $d\emptyset_m/dt$ is in Wb/s.

The minus sign indicates that the induced emf opposes the change which produces it, as stated generally in Lenz's law.

Lenz's law stated that the direction of any magnetic induction effect is such as to oppose the cause of the effect. For example, if the flux is increasing through a coil, the current produced by the induced emf will generate a flux that tends to cancel the increasing flux. Or, if the flux is decreasing through the coil, that current will produce a flux that tends to restore the decreasing flux. Lenz's law is a consequence of conservation of energy.

Motional EMF

When a conductor moves through a magnetic field so as to cut field lines, an induced emf will exist in it, in accordance with Faraday's law. But, in this case

$$|\varepsilon| = \frac{d\emptyset_m}{dt}$$

The symbol $|\varepsilon|$ means that we are concerned only with the magnitude of the average induced emf.

The induced emf in a straight conductor of length L moving with velocity $\vec{V}$ perpendicular to a field $\underline{B}$ is given by

$$|\varepsilon| = BLv$$

Where $\vec{B}$, $\vec{V}$ and the wire must be mutually perpendicular.

In this case, Lenz's law still tells us that the induced emf opposes the process. But now the opposition is produced by way of the force exerted by the magnetic field on the induced current in the conductor. The current direction must be such that the force opposes the motion of the conductor knowing the current direction, we also know the direction of ε.

Sources of Magnetic Fields

Magnetic fields are produced by moving charges, and of course, that includes electric currents. The constant μ_0= 4π X 10^{-7}T.m/A is called the permeability of free space. It is assumed that the surrounding material is a vacuum of air.

Bohr Magneton

We picture the election (mass m_1 charge – e) as moving in a circular orbit with radius r and speed V. This moving charge is equivalent to a loop. We know that magnetic dipole moment μ is given by μ =IA; for the orbiting electron the area of the loop is A=πr^2. To find the current associated with the electron, the orbital period T is the circumference divided by the electron speed: T = 2 πr/v the equivalent current I is the magnitude e of the electron charge divided by the orbital period T:

$$I = e/T = eV/2\pi r$$

Magnetic moment μ=IA, $\mu=\frac{eV}{2\pi r}(\pi r^2)= eVr/2$

For a particle moving in a circular path, the angular momentum L is L=mvr,

μ = eL/2m

But atomic angular momentum is quantized; its component in a particular direction is always an integer multiple of h/2π where h is planek's constant.

h= 6.626 X 10^{-34} Js

$$\mu=\frac{e}{2m}(h/2\pi)= eh/4\pi m$$

This quantity is called the Bohr magneton, denoted by μ_B. Its numerical value is

μ_B = 9.274 X 10^{-24} Am^2

Paramagnetism

Some atoms have a net magnetic moment that is of the order of μ_B and when placed in magnetic field, the field exerts a torque on each magnetic moment

$\vec{\tau} = \vec{\mu} \times \vec{B}$

This torque tends to align the magnetic moments with the field. In this position, the directions of the current loops are such as to add to the externally applied magnetic field. B field produced by a current loop is proportional to the loop's magnetic dipole moment. In the same way, the additional $\vec{B}$ field produced by microscopic electron current loops is proportional to the total magnetic moment μ_{total} per unit volume V in the material. This vector quantity is called the magnetization of the material, denoted by $\vec{M}$

$$\vec{M} = \vec{\mu}_{total}/V$$

The additional magnetic field due to magnetization of the material turns out to be equal simply to $\mu_0\vec{M}$, where μ_0 is the permeability of free space. When such material magnetic field B in the material is

$$\vec{B} = \vec{B_0} + \mu_0 \vec{M}$$

$\vec{B_0}$ is the field caused by the current in the conductor, magnetization $\vec{M}$ is magnetic moment per unit volume.

A material showing the behaviour just described is said to be paramagnetic. This shows that the magnetic field at any point in such material is greater by a dimensionless factor k_m, called the relative permeability of the material, than it would be if the material were placed by vacuum. The value k_m is varies for different materials; for common paramagnetic solids and liquids at room temperature, km typically ranges from 1.00001 To 1.003. For a particular material,

The permeability of such material is $\mu = k_m \mu_0$.

The amount by which the relative permeability differ from unity is called the magnetic susceptibility, denoted by $\mathcal{X}_m$

$$\mathcal{X}_m = k_m - 1$$

Paramagnetic susceptibility always decreases with increasing temperature. In many cases it is inversely propotional to the absolute temperature T, and the magnetization M can be expressed as $M = C^B/_T$

This relationship is called curie's law and the quantity C is a constant, different for different materials, called the curie constant.

Diamagnetism

In some materials the total magnetic moment of all the atomic current loops is zero when no magnetic field is present. But even those materials have magnetic effects because an external field alters electron motions within the atoms, causing additional current loops and induced magnetic dipoles. In this case, the additional field caused by these current loops is always opposite in direction to that of the external field. Such materials are said to be diamagnetic. They always have negative susceptibility, and their relative permeability km is slightly less than unity, typically of the order of 0.99990 to 0.99999 for solids and liquids. Diamagnetic susceptibilities are very nearly temperature independent.

Ferromagnetism

Ferromagnetic materials include iron, nickel, cobalt and all alloys containing these elements. In these materials strong interactions between atomic magnetic moments cause them to line up parallel to each other in regions called magnetic domains, even when no external field is present.

When there is no externally applied field, the domain magnetizations are randomly oriented. But when a field caused by external currents is present, the domains tend to orient themselves parallel to the field. The domain boundaries also shift; the domains that are magnetized in other direction grow, and those that are magnetized in other directions shrink. Because the total magnetic moment of a domain may be thousands of Bohr magnetons, the torques that tend to align the domains with an external field are much stronger than occur with paramagnetic materials. The relative permeability km is much larger than unity, typically of the order of 1000 to 100,000

Hysteresis

For many ferromagnetic materials, the relationship of magnetization to external magnetic field is different when the external field is increasing from when it is decreasing. The figure below shows this relationship for such materials when the material is magnetized to saturation and then the external field is reduced to zero, some magnetization remains. This behavior is characteristic of permanent magnets, which retain most of their saturation magnetization when the magnetizing field is removed. To reduce the magnetization to zero requires a magnetic field in the reverse direction.

This behavior is called hysterics, and the curve below is called hysteresis loop. Magnetizing and demagnetizing a material that has hysteresis involve the dissipation of energy, and the temperature of the material increases during such a process.

Magnetization M

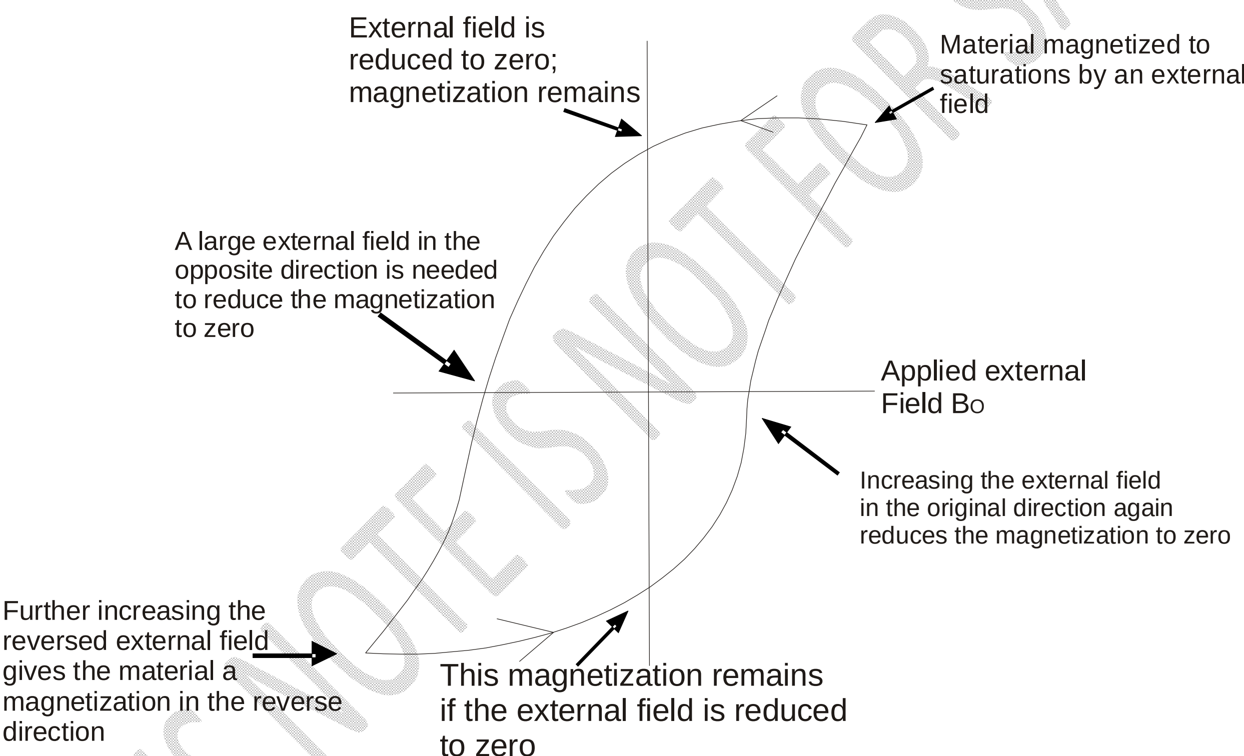


Fig.: Hysteresis loop

A.C. Circuits

The emf generated by a rotating coil in a magnetic field has a sinusoidal graph which is called an ac voltage because there is a reversal of polarity (i.e., the voltage changes sign). We use the term ac source for any device that supplies a sinusoidal varying voltage (potential difference) V or current i. The sinusoidal voltage might be described by a function such as

$$V = V_0 \cos wt$$

V is the instantaneous potential difference

V_0 is the maximum potential difference called the voltage amplitude.

w is the angular frequency = $2\pi f$

similarly, a sinusoidal current might be described as

$$i = I_0 \cos wt$$

i is the instantaneous current

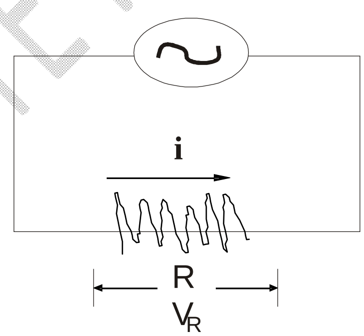
I_0 is the maximum current or current amplitude. The frequency f of the voltage is related to its period T by $T = {}^1/_F$

Meters for use in ac circuits read the effective, or root mean square (rms), values of the current and voltage. These values are always positive and are related to the amplitudes of the instantaneous sinusoidal values through

$V_{rms} = V_0 \ / \ \sqrt{2} = 0.707V_0$

$i_{rms} = I_0 \ / \ \sqrt{2} = 0.707I_0$

Resistor in an ac circuit is given by V = IR



$i = I \cos wt$
$V_R = iR = (IR) \cos wt$

V_R is the instantaneous voltage

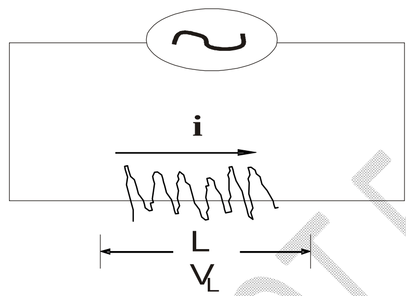
$V_m = IR$

V_m is the voltage amplitude or maximum voltage

Inductor in an a.c Circuit (L-Circuit)

This is an a.c circuit with only a pure inductor with self-inductance L and zero resistance. Let the current be I = I cos wt, no resistance, there is a potential difference V_L between the ends of the inductor terminal because the current varies with time, giving rise to a self-induced emf. The induced emf in the direction of i is directed to the left to oppose the increase in current.

Thus $V_L = +L\ {}^{di}/_{dt}$



$$V_L = L\frac{di}{dt} = L\frac{d}{dt}\ (I \cos wt) = - IwL \sin wt.$$

But cos $(A+90^0)$ = - sin A.

$$V_L = IwL \cos (wt + 90^0)$$

Thus if the current I in a circuit is i = I cos wt and the voltage V of one point with respect to another is

$$V = V\cos (wt + \theta)$$

∅ is the phase angle; it gives the phase of the voltage relative to the current. For a pure resistor, ∅ = 0, and for a pure inductor, ∅ = 90^0. The amplitude V_L of the inductor voltage is $V_L = IwL$

Inductive reactance X_L of an inductor is $X_L = WL$

$V_L = IX_L$ (amplitude of voltage across an inductor).

The inductive reactance X_L is a description of the self-induced emf that opposes any change in the current through the induction.

Capacitor in an ac Circuit (C-Circuit)

This is an a.c circuit with only a capacitor with capacitance C to the source, producing a current i = I cos wt through the capacitor. To find the instantaneous voltage V_C across the capacitor, we first let q denote the charge on the left-hand plate of the capacitor and –q is the charge on the right-hand plate. The current i is related to q by i = $\frac{dq}{dt}$

Positive current corresponds to an increasing change on the left hand capacitor plan. Then i = $\frac{dq}{dt}$ = I cos wt

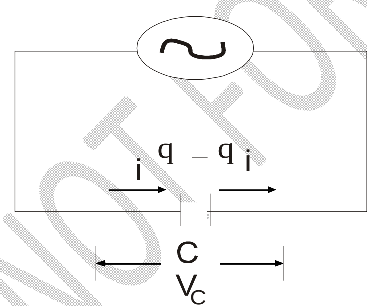
Integrating, q = i = $\frac{I}{w}$ sin wt

q = CV_C

V_C = i = $\frac{I}{wC}$ sin wt.

But cos (A - 90^0) = sin A

V_C = $\frac{I}{wC}$ cos (wt - 90^0)



The instantaneous current i is equal to the rate of change $\frac{dq}{dt}$ of the capacitor charge q, since q = CV_C, i is also proportional to the rate of change of voltage. This corresponds to a phase angle Ø = 90^0. This cosine function has a "late start" of 90^0 compared with the current i = I cos wt.

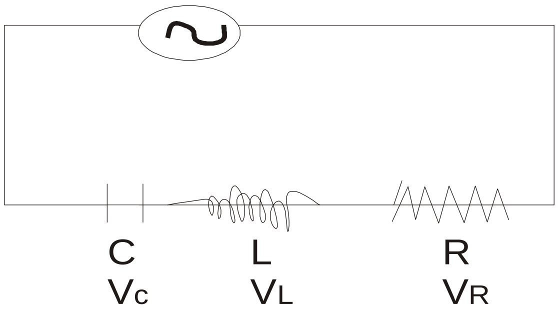
The maximum voltage V_m is V_m = $\frac{I}{wC}$

Capacitive reactance of the capacitor is X_c = $\frac{I}{wC}$

V_c = I X_c (amplitude of voltage across a capacitor).

The greater the capacitance and the higher the frequency, the slower the capacitive reactance X_c. Capacitors tend to pass high frequency current and to block low – frequency currents and dc, just the opposite of indicator.

L – R – C Series Circuit



The Potential difference acreoss a resistor, maximum voltage V_R= IR.

The Voltage across the inductor, its voltage amplitude V_L= I X_L.

The voltage across a capacitor, its voltage amplitude V_C= I X_C.

$$V=\sqrt{V_R^2 + (V_L - V_C)^2} \quad = \sqrt{(IR)^2 + (I X_L - I X_C)^2}$$

$$V= I \sqrt{R^2 + (X_L - X_C)^2}$$

Impedance Z of an ac circuit is the ratio of the voltage amplitude across the circuit to the current amplitude in the circuit.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$V= I Z$$

$$Z = \sqrt{R^2 + (W_L - (\frac{1}{WC})^2}$$

$$\text{Phase= } \tan \emptyset = \frac{V_L - V_C}{V_R} = I \frac{(X_L - X_C)}{IR} = \frac{X_L - X_C}{R}$$

$$\emptyset = \tan^{-1} (\frac{X_L - X_C}{R}) \qquad \emptyset = \frac{R}{Z}$$

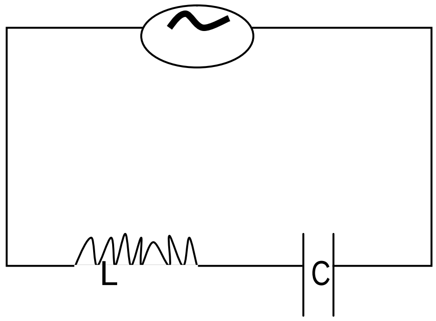
$$\tan \emptyset = \frac{WL - \frac{1}{WC}}{R}$$

$$\emptyset = \tan^{-1} (\frac{WL - \frac{1}{WC}}{R})$$

If the current of an L-R-C circuit is i= I cos wt, then the source voltage is

V=vcos (wt + ∅).

For an L-C circuit

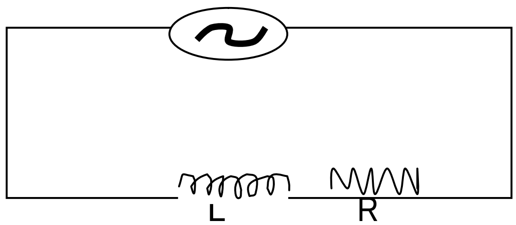


V= I $\sqrt{(X_2 - X_C)^2}$

Ƶ = $\sqrt{(WL - (\frac{1}{WC})^2}$

Ƶ =$\sqrt{(X_L - X_C)^2}$

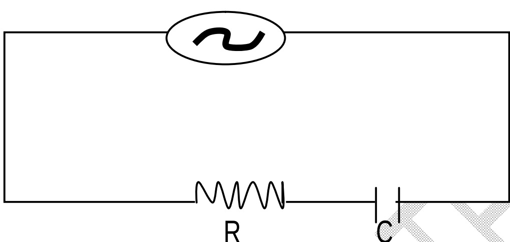
For an L-R circuit



V= I $\sqrt{R^2 + X_L^2}$

Ƶ= $\sqrt{R^2 + (wL)^2}$
Ƶ= $\sqrt{R^2 + X_L^2}$

For an R-C circuit



V= I $\sqrt{R^2 + (-X_L)^2}$
V = I $\sqrt{R^2 + X_C^2}$

Ƶ = $\sqrt{R^2 + (\frac{1}{WC})^2}$

Ƶ= $\sqrt{R^2 + (X_C)^2}$

For an ac circuit, $V_{rms} = \frac{V}{\sqrt{2}}$

$I_{rms} = \frac{I}{\sqrt{2}}$

$V_{rms} = I_{rms}$ Ƶ

Resonance occurs in a series R-L-C circuit when X_L=X_C, under this condition, Ƶ =R is minimum, so that I is maximum for a given value of V.

X_L=X_C, wL=$\frac{1}{w_0 C}$, w_o= $\frac{1}{\sqrt{LC}}$

The resonance frequency f_0 is $\frac{w_0}{2\pi}$, $f_0 = \frac{1}{2\pi\sqrt{LC}}$

Power in an a.c Circuit

For an ac circuit with instantaneous current I and current amplitude I and instantaneous voltage V with voltage amplitude V. The instantaneous power P is P=vi

Power in a Resistor

For a pure resistor R, then voltage and current are in phase
The average power $P_{av} = \frac{1}{2}VI$

$$P_{av} = \frac{V}{\sqrt{2}} \cdot \frac{I}{\sqrt{2}} = V_{rms}\, I_{rms}$$

$$P_{av} = I_{rms}^2 R = \frac{V_{rms}^2}{R} = V_{rms}\, I_{rms}$$

Power in an Inductor

For a pure inductor L, voltage leads the current by 90°.The power is positive half the time and negative the other half, and the average power is zero. When P is positive, energy is being supplied to set up the magnetic field in the inductor; when P is negative, the field is collapsing and the inductor is returning energy to the source. The net energy transfer over one cycle is zero.

Power in a Capacitor

In a pure capacitor C, the voltage lags the current by 90°. The average power is also zero. Energy is supplied to charge the capacitor and is returned to the source when the capacitor discharges the net energy transfer over one cycle is again zero.

Power in a General AC Circuit

In any ac circuit, with any combination of resistors, capacitors, and inductors, the voltage V across the entire circuit has some phase angle Ø with respect to the current i. The instantaneous power P is

P= Vi = [V cos (wt +Ø)] [I cos wt]

P=[V(cos wt cos Ø- sin wt sin Ø)][Icos wt]

=VIcos Ø $\cos^2$ wt – VI sin Ø cos wt sin wt)

The average value of $\cos^2 wt$ cover one cycle) is ½ and the average value of cos wt sin wt is zero. Because it is equal to ½ VIcos Ø= V|rms Irms cos Ø

The factor cos Ø is called the power factor of the circuit.

Exercises

1. In an a. c. circuit, the peak value of the potential difference is 180V.what is the instantaneous p. d when it has reached $1/8^{th}$ of a cycle
(90.2V)

2. In a series L-C circuit, the inductance and capacitance are 0.5henry and 2.0microfarad respectively. Calculate the resonant frequency of the circuit
(50.3Hz)

3. The current in a series R-L-C circuit attains its maximum value when the
(A) impedance is greater than the capacitive reactance (B) inductance reactance is equal to the capacitive reactance (C) inductance reactance is greater than the capacitive reactance (D) capacitive reactance is less than the resistance

4. An inductor of inductance 10H carries a current of 0.2A.Calculate the energy stored in the inductor
(2.0J)

5. Calculate the inductance of an inductor whose reactance is one ohm at 50Hz
(3.18×10^{-3}H)

6. The parts of bar magnet at which the magnetic effect is strongest are called the
(A) Poles (B) Neutral points (C) Magnetic declination (D) Magnetic meridian

7. Which of the following modes is the most economical method of transmitting electric power over long distances?
(A) Alternating current at low voltage and high current (B) alternating current at high voltage and high current (C) alternating current at high voltage and low current (D) direct current at low voltage and high current

8. An alternating current with a peak value of 5A passes through a resistor of resistance 10.0ohms.Calculate the rate at which energy is dissipated in the resistor (A) 250.0W (B) 125.0W (C)50.0W (D)35.4W

9. In a R-C circuit (A) I_{rms} leads V_{rms} by 60 (B) I_{rms} lag V_{rms} by 60 (C) V_{rms} lags I_{rms} by 90 (D) V_{rms} leads I_{rms} by 90

10. The energy E stored in an inductor of inductance L when current I passes through it is given by the equation
($E = \frac{1}{2}L^2I$)

11. An ammeter connected to an a.c. circuit records 5.5A.The peak current in the circuit is:
(7.8A)