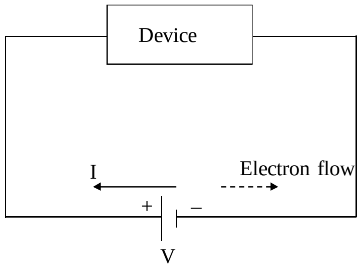
Ohm's Law



The ratio V/I is a constant, where V is the voltage applied across a piece of material (device) such as a wire and I is the resulting current through the material.

$$\frac{V}{I} = R = constant \quad \text{or} \quad V = IR$$

R is the resistance of the piece of material and the unit is in ohm (Ω). This implies that a wire or an electrical device (called resistor) offers resistance to the flow of charges.

Ohm's Law

Symbol 

In an electrical circuit, straight line (-) represents an ideal conducting wire, or one with a negligible resistance.

<u>Resistance and Resistivity</u>

Resistance R of a piece of material is proportional to its length (L) and inversely proportional to its cross-sectional area (A)

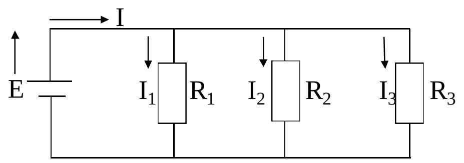
$$R = \rho \frac{L}{A}$$

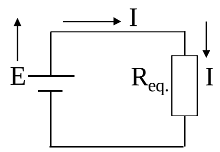
ρ is the proportionality constant known as the resistivity of the material.

ρ is an inherent property of a material just like density. R depends on resistivity and geometry of the material.

Parallel and Series Arrangement of Resistors

Parallel Arrangement:





For resistors in parallel, the same voltage drops across them (or passes through them) i.e. Total current $I = I_1 + I_2 + I_3$

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

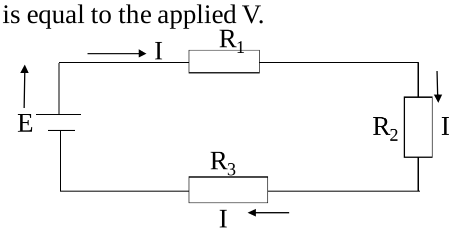
$$I = \frac{V}{R_{Eq.}}, \quad \frac{1}{R_{Eq.}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

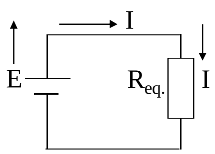
$$\frac{1}{R_{Eq.}} = \sum_{j=1}^{n} \frac{1}{R_j}, \quad \textit{for n resistors in parallel}$$

Parallel and Series Arrangement of Resistors

Series Arrangement:

When a potential difference V is applied across resistors connected in series, the resistors have identical current I. The sum of the potential differences across the resistors is equal to the applied V.





$$V = I \cdot R_1 + I \cdot R_2 + I \cdot R_3 = I(R_1 + R_2 + R_3)$$

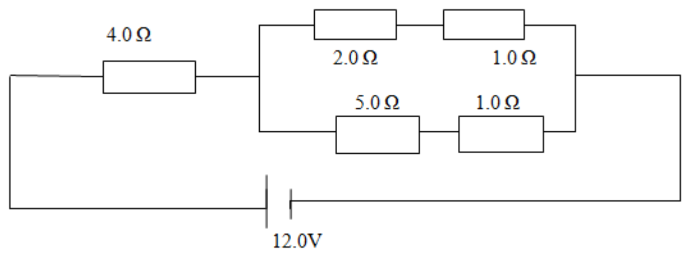
$$I = \frac{V}{R_1 + R_2 + R_3} = \frac{V}{R_{Eq.}}$$

$$R_{Eq.} = \sum_{j=1}^{n} R_j, \quad \textit{for n resistors in series}$$

Examples

Q1. Two resistors, 16.0 Ω and 8.0 Ω, are connected in series across a 12.0V battery. What is the voltage across each resistor.

Ans: V across 16 Ω = 8V, V across 8 Ω = 4 V



Q2. Determine the power dissipated in the 2.0 Ω resistor in the circuit above (Resistive dissipation)

Ans: power dissipated = 2.667 W

Variation of Resistance with Temperature

The ions in a metal lattice vibrate more quickly as the temperature increases. This makes it more likely that an electron will interact with an ion and loses energy.

Therefore, the resistance of a metallic conductor increases, and for pure metals it increases linearly with temperature.

$$R_\theta = R_o(1 + \alpha_R(T - T_o))$$

where

R_θ – R at temp. T (Ω)

R_o – R at temp. T_o (Ω)

α_R – is the temp. coefficient of resistance (°C^{-1})

Variation of Resistivity with Temperature

Similarly

$$\rho = \rho_o(1+\alpha_\rho(T - T_o))$$

Conductivity

$$\sigma = \frac{1}{\rho} \qquad (\Omega.m)^{-1}$$

The unit is written ad mhos per meter

Examples:

- A copper wire and aluminum wire have the same length. Obtain the ratio of diameter of aluminum to that of copper if the resistance of copper is twice that of aluminum and the resistivity of copper ρ_c = 1.72 x 10^{-8} Ωm and that of aluminum ρ_a = 1.72 x 10^{-8} Ωm.
  Ans: 9:5
- Calculate the resistance per meter length of constantan wire of diameter 0.4 mm. What length of constantan would be required to make a resistor of resistance of 1.5 Ω.
  $\rho_{constantan}$ = 4.70 x 10^{-5} Ωm
  Ans: R/L = 374.0 Ωm^{-1} Length = 0.004 m
- A wire 4.0 m long and 6.00 mm diameter has a resistance of 15.0 mΩ. A potential difference of 23.0 V is applied between the ends of the wire, (a) what is the current through the wire. (b) calculate the rsistivity of the wire material.
  Ans: a = 1.53 x10^3 A b = 1.06 x 10^{-7} Ωm
- A coil is formed by winding 250 turns of insulated 16-gauge copper wire (diameter = 1.3 mm) in a single layer on a cylindrical form of radius 12 cm. What is the resistance of the coil? (Neglect the thickness of the insulation and ρ_c = 1.72 x 10^{-8} Ωm)
  Ans: 2.4 Ω

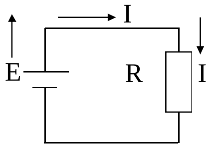
Examples

1. A digital thermometer uses a thermistor as the temperature sensing element. A thermistor is a kind of semiconductor and has a large negative temp. coefficient of resistivity α. Suppose $\alpha = -0.06$ ($°C^{-1}$) for the thermistor in a digital Thermometer used to measure the temp. of a sick patient. The resistance of the thermistor decreases by 15% relative to its value at the normal body temp. of 37.0 °C. What is the patient's temp.?
   Ans: 39.5 °C

2. A wire has a resistance of 21 Ω. It is then melted down, and from the metal a new wire is mode that is three times as long as the original wire. What is the resistance of the new wire?
   Ans: 189 Ω

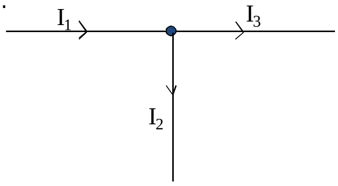
Kirchhoff's Rules

Electric circuit is the path through which charge can flow.



Two basic rules that apply in all electric circuit are called Kirchhoff's laws:

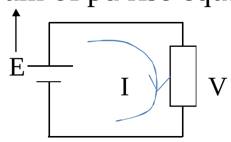
Kirchhoff's 1st law (Junction Rule)
Total current arriving at a junction in a circuit must equal to the total current leaving the junction.



$$I_1 = I_2 + I_3$$

Kirchhoff's Rules

Second Law (Loop Rule / Voltage law):
The sum of potential difference round any closed loop in a circuit must be zero (or the sum of pd rise equal the sum of pd drop)

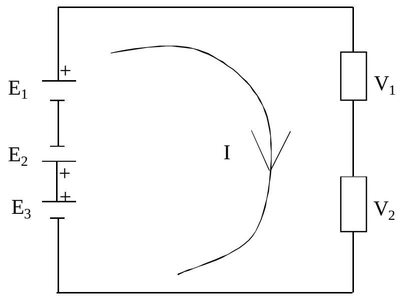


$$E - V = 0$$

Resistance Rule: For a move through a resistor in the direction of current, the change in potential is *–I.R*, in the opposite direction it is *+I.R*.

EMF Rule: For a move through an ideal *emf* device in the direction of *emf* arrow, the change in potential is +E; in the opposite direction it is –E.

Kirchhoff's Rules: Examples

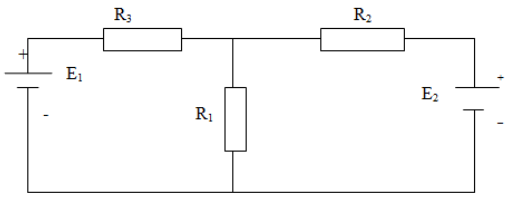


$$E_1 - E_2 + E_3 - V_1 - V_2 = 0$$
$$E_1 - E_2 + E_3 = I \cdot R_1 + I \cdot R_2$$
$$I = \frac{E_1 - E_2 + E_3}{R_1 + R_2}$$

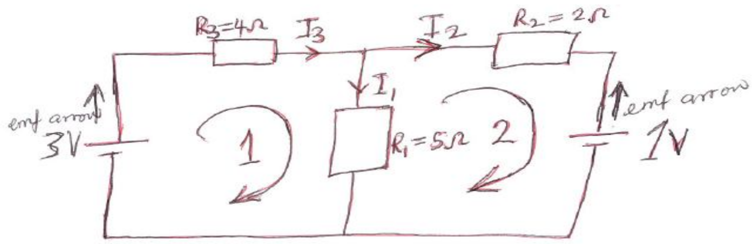
Kirchhoff's Rules: Examples

Q1. E_1 = 3.0V, E_2 = 1.00V, R_1 = 5.0 Ω, R_2 = 2.0 Ω, R_3 = 4.0 Ω and both batteries are ideal. What is the rate at which energy is dissipated in (a) R_1 (b) R_2 (c) R_3

Ans: P_1 = 0.35 W P_2 =0.05 W & P_3 = 0.71 W



Solution



Using junction's law, $I_3 = I_1 + I_2$ — (1)

From loop 1, moving clockwise through the loop

$3 - 4I_3 - 5I_1 = 0$ — (2)

From loop 2, moving clockwise also through the loop

$-1 - 2I_2 + 5I_1 = 0$

$\Rightarrow 5I_1 - 2I_2 = 1$ — (3)

From eqn 2, replace I_3 with $(I_1 + I_2)$ and re-arrange

$$9I_1 + 4I_2 = 3 \quad \text{---} (4)$$

Put (3) and (4) together and solve simultaneously

$$2 \times \text{eqn } (3) \quad 10I_1 - 4I_2 = 2$$

$$1 \times \text{eqn } (4) \quad 9I_1 + 4I_2 = 3$$

add

$$19I_1 = 5$$

$$I_1 = 5/19 \text{ A} = 0.263\text{A} \quad (5)$$

put (5) in 4 to obtain I_2

$$9\left(\frac{5}{19}\right) + 4I_2 = 3$$

2.368

$$I_2 = \frac{3 - 2.368}{4} = 0.158\text{A}$$

Put the values of I_1 and I_2 in eqn 1

$$I_3 = 0.263 + 0.158$$

$$= 0.421\text{ A}.$$

Rate at which energy is dissipated is the same as Power dissipated and,

$$\text{Power } P = I^2R$$

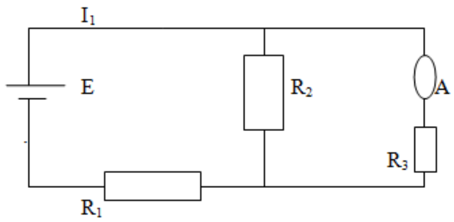
$$P_{R1} = (0.263)^2 \times 5 = 0.35\text{ W}$$

$$P_{R2} = (0.158)^2 \times 2 = 0.05\text{ W}$$

$$P_{R3} = (0.421)^2 \times 4 = 0.71\text{ W}$$

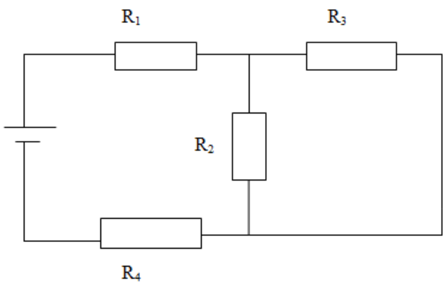
Kirchhoff's Rules: Examples

Q3. Determine what the ammeter will read, assuming E = 5.0V (for the ideal battery), R_1= 2.0 Ω, R_2 = 4.0 Ω and R_3 = 6.0 Ω.   Ans: 0.454 A



Q4. R_1 = 20 Ω, R_2 = 20 Ω, R_3 = 30 Ω, R_4 = 8 Ω and E = 12V. What is the current through R_1.

Ans: 0.3 A



Electrical Energy & Power

A battery can deliver power to an electric circuit. Power is the amount of work exerted over an interval of time.

$$P = \frac{W}{\Delta t}$$

Work is equal to the change of potential energy

$$W = \Delta PE = q\Delta V$$

$$P = q\frac{\Delta V}{\Delta t}$$

If we have a current of charges Δq across a voltage difference ΔV, we can re-write

$$P = \Delta q \frac{\Delta V}{\Delta t} = \Delta V \frac{\Delta q}{\Delta t} = \Delta V \cdot I$$

$$P = I \cdot \Delta V \quad or \quad P = I \cdot V$$

Power is measured in Watt (W). $1W = A.V$

Electrical Energy & Power

Using Ohm's law, $P = I^2 R = \frac{V^2}{R}$

The power delivered to a conductor of resistance R is often referred to as an I^2R loss.
Electrical Energy E = P . t = IVt

Ohmic device: a device that follows Ohm's law for all voltages across it is called an Ohmic device (i.e. the resistance of the device is independent of the magnitude and polarity of the applied potential difference), and the resistance is said to have a constant value (static resistance).
Examples are wire, electric stove heating element or a resistor, incandescent light bulb etc.

Electrical Energy & Power

Non-Ohmic device: a device that behaves in a way that is not described by Ohm's law (i.e. the resistance is not constant but changes in a way that depends on the voltage across it).

Examples are tungsten filament (bulb), diode, thermistor etc