

Wk. 7

+ moving coil galvanometer

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Question

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in either
margin

Electromagnetic Induction

If a closed conducting loop is placed or immersed in a magnetic field and current is sent through it, force due to magnetic field create a torque which turns or rotates the loop.

Current loop + magnetic field \Rightarrow torque - - - 1

But if current is off and the loop is turned by hand experiment shows that current will appear in the loop:

torque + magnetic field \Rightarrow current - - - 2

The physical law on which eqns (1) & (2) depends is called Faraday's law of electromagnetic induction.

Faraday's Law of Induction

Faraday observed that an emf and a current can be induced in a loop by changing the amount of magnetic field passing through the loop. The amount of the magnetic field can be visualized in terms of the magnetic field lines passing through the loop.

The amount of magnetic field that passes through any surface of area A (magnetic flux) is given as

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

Here, A is the area enclosed by the loop placed in a magnetic field \vec{B} .

As explained earlier, if the magnetic field is uniform the eqn reduces to

$$\Phi_B = B \cdot A$$

Faraday's law can be stated using the notion of magnetic flux as:

The magnitude of the emf \mathcal{E} induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through the loop changes with time.



1.2

$$\mathcal{E} = - \frac{d\Phi_B}{dt} \quad \text{--- Faraday's Law ---}$$

-VE sign indicates the direction of emf (\mathcal{E}) which is opposite to the motion that produces it.

If the ^{loop} has N number of turns, an induced emf appears in every turn and the total emf induced in the coil is the sum of the individual induced emfs.

$$\mathcal{E} = - N \frac{d\Phi}{dt}$$

Q. A certain coil of wire consists of 500 circular loops of radius 4 cm. It is placed b/w. the poles of a large electro-magnet, where the magnetic field is uniform, perpendicular to the plane of the coil, and increasing at a rate of $0.2 \text{ T} \cdot \text{s}^{-1}$. What is the magnitude of the resulting induced emf?

Q. A rectangular coil of wire having 10 turns with dimensions of 20 cm x 30 cm rotates at a constant speed of 600 rpm in a magnetic field 0.10 T . The axis of rotation is \perp to the field. Find the maximum emf produced.

Q

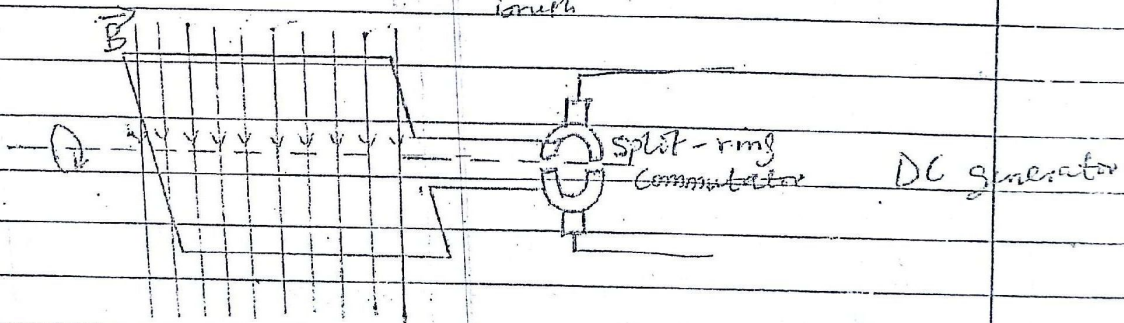
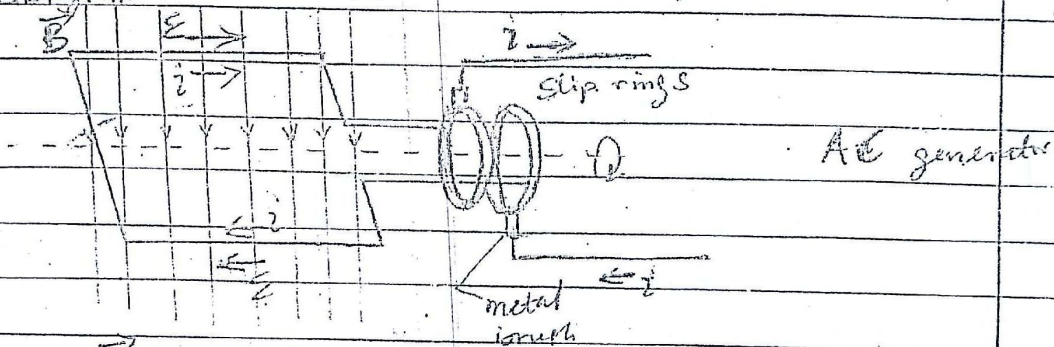


AC and DC generator

A generator is a device that convert the mechanical energy to electrical energy. A generator is basically a moving (rotating) coil of wire in a magnetic field. A mechanical force rotates the ~~supplies~~ a turning effect ^{or} force (torque) on the coil. ~~find from the principle~~ ^{Faraday's} ~~of~~ electromagnetic induction.

torque + magnetic field \Rightarrow current.

~~mechanical diagram~~



The emf induced or emf generated according to Faraday's law

$$\mathcal{E} = - \frac{d\phi}{dt}$$

where $\phi = BA \cos \theta$

where θ is the angle btw \vec{B} and $d\vec{A}$

but $\theta = \text{angular velocity} \times \text{time} = \omega t$

$$\phi = BA \cos \omega t$$

$$\mathcal{E} = - N \frac{d(BA \cos \omega t)}{dt}$$

$$\text{Emf} = NBA \omega \sin \omega t$$



~~Lenz's Law~~

$$\mathcal{E}_{\text{ind}} = + N \frac{d\Phi}{dt}$$

Q1. A circular turn of wire of radius r rotates with an angular velocity of 1800 rpm about a diameter which is \perp to a uniform magnetic field of flux density 0.5 Wb/m^2 .
 (a) What is the instantaneous induced \mathcal{E} in the turn when the plane of the turn makes an angle of 30° with the direction of the flux? (b) What is the angle b/w the plane of the turn and the flux, when the instantaneous \mathcal{E} had the same value as the average \mathcal{E} for a half cycle?

$$I_{\text{av}} = \frac{\omega}{2\pi} \int_{-\pi/2}^{+\pi/2} I_m \cos \omega t dt$$

$\sin(\theta) = -\cos(\theta)$

$$I_{\text{av}} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f(t) dt$$

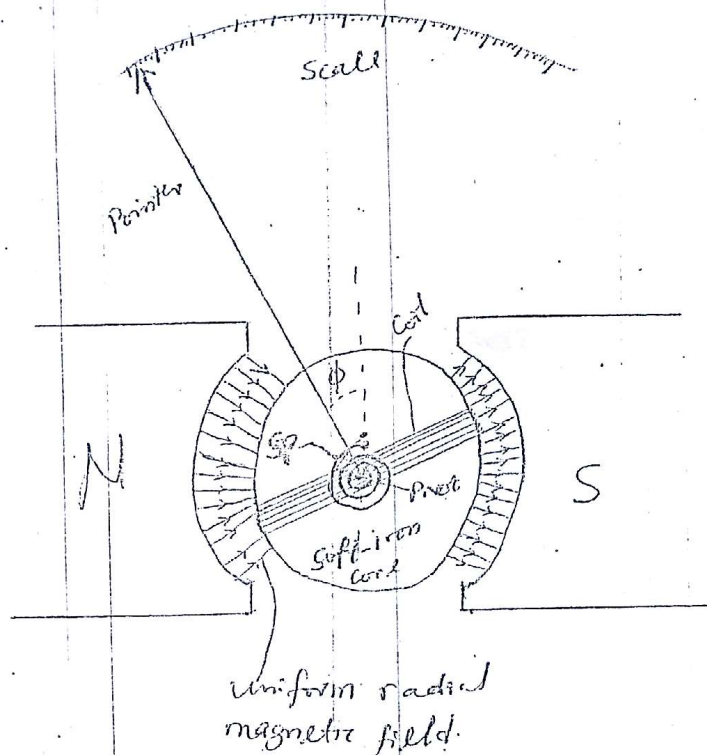
$$\mathcal{E} = \mathcal{E}_m \cos \omega t$$

$$\frac{\mathcal{E}_m}{\pi}$$

MOVING COIL GALVANOMETER

(16)

A galvanometer is an instrument used to detect small current through a circuit. The principle of operation of a moving coil galvanometer is based on Torque that a current loop will experience when immersed in a magnetic field \vec{B} .



The diagram of a moving coil galvanometer is shown above.

A moving coil attached to a spring S_p is mounted so that it can rotate about an axis in a uniform radial magnetic field. For any orientation of the coil, the net magnetic field through the coil is \perp to the normal vector of the coil (and thus parallel to the plane of the coil). A Spring S_p provides a counter torque that balances the magnetic torque, so that a given steady current i in the coil results in a steady angular deflection ϕ . The greater the current is, the greater the deflection ϕ is, and thus greater the torque required of the Spring is. The pointer attached to the Spring on the coil

states with the coil and from the scale, the deflection can be read.

The Spring S_p balances the magnetic torque \Rightarrow

$$NiAB \sin \theta = k \phi$$

But θ is 90° since the field is always \perp to the normal vector of the coil.

$$NiAB = k \phi$$

Here k is the torsional constant of the spring and ϕ the angular deflection. If k of the spring is known, for any deflection ϕ

$$i = \frac{k \phi}{NAB}$$

$\frac{k}{NAB}$ is a constant i.e. $i \propto \phi$. The deflection

can be graduated on the scale to give the current through the coil.

12. A coil of dimension 2.1cm by 1.2cm has 250 turns. The coil is mounted in a uniform radial magnetic field with $B = 0.23T$. If a current of 100mA produces an angular deflection of 25° , what is the torsional constant k of the spring.

12. The coil of a certain galvanometer has a resistance of 75.3Ω . Its needle shows a full-scale deflection when a current of 162mA passes through the coil. (a) Determine the value of the auxiliary resistance required to convert the galvanometer to a voltmeter that reads 1.0V at full-scale deflection. How should this resistance be connected? (b) Determine the value of the auxiliary resistance required to convert the galvanometer to an ammeter that reads 50.0mA at full-scale deflection. How should this resistance be

NKS

Magnetic materials

①

This are materials which when ^{current is} introduced into an external magnetic field, ^{causes it to} change so that they also become source of magnetic field themselves.

The total magnetic induction in this case is the sum of the inductions of the external magnetic field and the magnetic field generated by the magnet. The phenomenon of ~~induction~~ the change in the state of a magnetic material under the influence of an external magnetic field as a result of which the material itself becomes a source of a magnetic field is called the magnetization of the magnetic material.

All materials regarding their magnetic behaviour can be classified into three categories, namely - Paramagnetic, Diamagnetic and Ferro-magnetic.

Paramagnetic materials

In these materials, the motion of the electrons in the molecules is such that the molecules will have a magnetic moment even in the absence of an external field, i.e. the molecules possess a permanent magnetic moment. Each molecule is a source of the external field. In the absence of the external field, the magnetic moments of different molecules are oriented randomly, so that the total magnetic induction of the field produced is equal to zero i.e. the magnetic induction of each molecule cancel

each other.

↳

When the material is introduced into an external magnetic field, the permanent magnetic moment of individual molecules are reoriented in the direction of the ~~field~~ magnetic induction of the field. The materials which when placed in a magnetic field acquire a feeble magnetization in the same direction as the applied field are called paramagnetic materials. These include, platinum, aluminium ~~mag~~ manganese, chromium, copper sulphate iron or nickel salt solution and crown glass.

Their properties are summarized as follows.

- i) When in a uniform magnetic field they rotate until their longest axes are parallel to the field.
- ii) When in a non-uniform magnetic field, experience an ~~an~~ attractive force towards the stronger part of the field.
- iii) When in a magnetic field, the magnetic induction B is more than the magnetising field H .
- iv) The susceptibility varies inversely as the absolute temperature.
- v) It acts like a diamagnetic when surrounded with more paramagnetic materials than themselves.

Diamagnetic Materials

(3)

When these materials are placed in a magnetic field acquires a feeble magnetisation in the direction opposite to that of the field. The materials includes - bismuth, antimony, water alcohol and hydrogen. They have the following properties.

- i) The materials set themselves at right angle to the field.
- ii) When in a non uniform field, move from the stronger part to the weaker part of the field.
- iii) When in a uniform magnetic field, the magnetic induction B is less than the magnetising field H .
- iv) The susceptibility has a small negative value and ~~does~~ ^{does} not vary with ~~the~~ temperature.
- v) ~~It~~ They reduces the flux density B .

Ferromagnetic material

The magnetization of these materials are considered to be a source of a magnetic field in the same way as the magnetization of dia and paramagnetic materials. The magnetization of these materials is retained even in the absence of the external magnetic field and the field generated by magnetization of these material exist independently. The materials acquires high degree of magnetization. the same way as the applied field.

These materials includes - Iron, nickel Cobalt and steel. They have the following properties.

i) They show the properties of paramagnetism to much higher degree and have permeability of the order of hundreds and thousands.

ii) The value of susceptibility is very high and positive.

iii) The susceptibility (I/H) is constant for small values of H , and ^{for} moderate values of H , the value of I increases rapidly with H and for large values attains a constant value. Hence the value of χ increases for moderate values of H and decreases for large values of H .

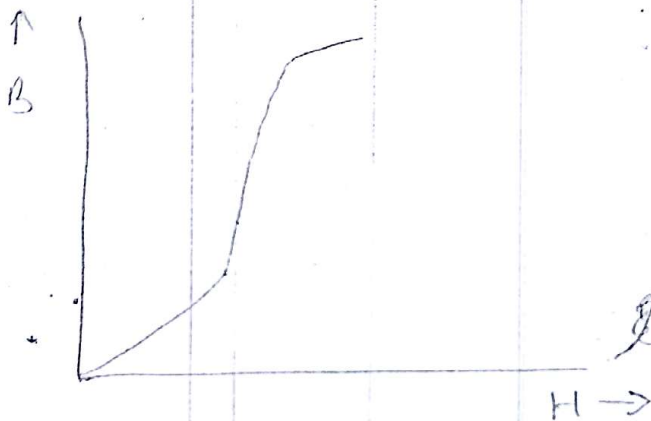
iv) The magnetic induction B changes with H in the same fashion as I does, except that B does not attain a constant value for large values of H .

v) As the temperature increases, the value of χ decreases. When the temperature reaches a certain ~~low~~ value, the material becomes paramagnetic material. This temperature is called Curie temperature. The Curie temperature is 1000°C , 770°C and 360°C for Iron, steel and nickel respectively.

Hysteresis

(5)

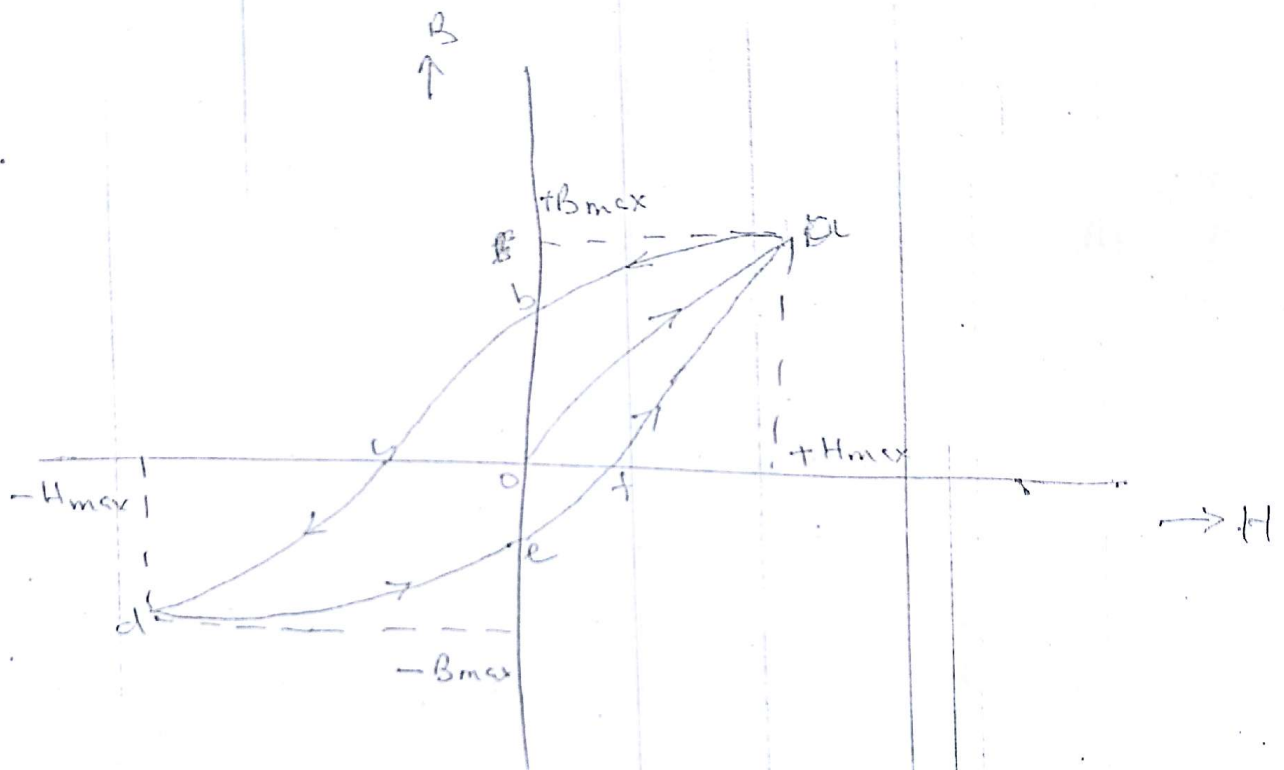
The variation of the flux density B with the magnetic field intensity H is not linear. When a graph of B is plotted against H , the result curve is obtained.



From the graph, we can see that, the curve rises rapidly at first, which indicates a very big change in the value of B for a corresponding small change in H . The slope of this curve gradually decreases, indicating a small change in B for a very large change in H . It will get to a point when the slope of the curve becomes constant which is known as the saturation point.

If a ferromagnetic material which is initially in its demagnetised state is made to undergo a cycle of magnetization in which H increases from zero to a maximum, then decreases to zero, then reversed and again taken to $-H_{max}$ and finally brought

back to zero. The variation of B with respect to H is as shown in the diagram below.



is called hysteresis

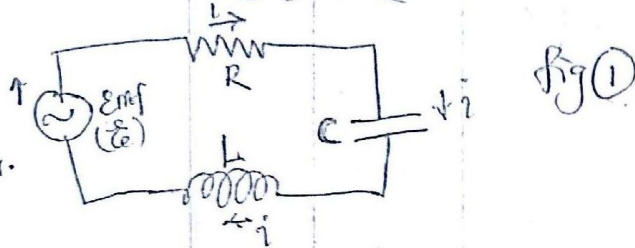
Increase the magnetising force H from zero to H_{max} and measure the corresponding values of flux density B and plot them in a $B-H$ graph, we obtain

from the graph, we obtain the curve oa , which is the normal magnetization curve. Now decrease the magnetising force H from H_{max} to zero and measure the corresponding values of B , the induction will not fall rapidly as it increases and fall back to b and not zero, thus having a curve ab and get back to oa . This shows that, when magnetising force is zero or removed, the iron is still magnet and the flux density is

is called residual magnetism. Now to reduce the residual magnetism, the magnetising force H is reversed and increased to $-H_{max}$ and the corresponding B value of reduce H to zero and further to H_{max} , the curve defo is obtained. The lagging of the flux density B with respect to magnetising force H is called hysteresis and the closed loop is known as hysteresis loop. (7)

Power in Alternating Current Circuits

In the RLC ccts, the source of energy is the alternating-current generator.



Eqn 4 for rms

Some of the energy that it provides is stored in the electric field in the capacitor, some is stored in the magnetic field in the inductor, and some is dissipated as thermal energy in the resistor. In steady-state operation which we assume the average energy stored in the capacitor and inductor together remains constant. The net transfer of energy is thus from the generator to the resistor, where electromagnetic energy is dissipated as thermal energy.

The instantaneous rate at which energy is dissipated in the resistor, however, can be written as;

$$P = i^2 R = [I \sin(\omega t - \phi)]^2 R = I^2 R \sin^2(\omega t - \phi) \quad \text{--- (1)}$$

The average rate at which energy is dissipated in the resistor, however is the average of eqn (1) over time.

Consider the graphs below;

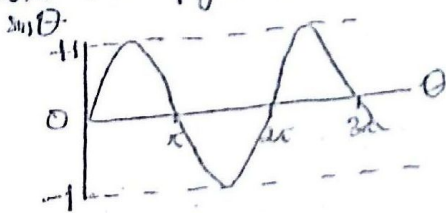


fig. (2)

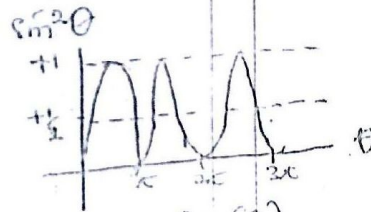


fig (3)

Over one complete cycle, the average value of $\sin^2 \theta$, where θ is any variable, is zero fig (2) but the average value of $\sin^2 \theta$ is $1/2$ fig (3). Note: (In fig (3), the shaded areas under the curve but above the horizontal line marked $1/2$ exactly fill in the unshaded spaces below that line). Thus, eqn (1) can be written as;

$$P_{avg} = \frac{I^2 R}{2} = \left(\frac{I}{\sqrt{2}}\right)^2 R \quad \text{--- 2}$$

where $\frac{I}{\sqrt{2}}$ is called the root-mean square or rms value of the current i

Eqn (2) can now rewrite as

$$I_{rms} = \frac{I}{\sqrt{2}} \quad \text{--- 3}$$

(4)

$$P_{avg} = I_{rms}^2 R \quad \text{--- 4} \quad P_{avg} = \text{average power}$$

Eq 4 looks much like $P = i^2 R$, the message is that if we know the rms current, we can compute the average rate of energy dissipation for alternating-current circuits just as for direct-current circuits.

We can also define rms values of voltages and emfs for alternating current circuits:

$$V_{rms} = \frac{V}{\sqrt{2}} \quad \text{and} \quad E_{rms} = \frac{E_m}{\sqrt{2}} \quad (\text{rms voltage; rms emf}) \quad \text{--- (5)}$$

Alternating-current instruments such as ammeters and voltmeters are usually calibrated to read I_{rms} , V_{rms} & E_{rms} . Thus, if you plug an alternating-current voltmeter into a household electrical outlet and it reads 120V, that is an rms voltage. The maximum value of $p.d.$ at the outlet is $\sqrt{2} \times (120V)$ or 170V.

Because the proportionality factor $\frac{1}{\sqrt{2}}$ in 3 and 5 is the same for all three variables, so we write $I = \frac{E_m}{Z}$ and $I = \frac{E_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}}$ as:

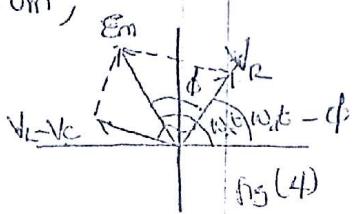
$$I_{rms} = \frac{E_{rms}}{Z} = \frac{E_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}} \quad \text{--- (6)}$$

and indeed, this is the form that we almost use.

We can use the relationship $I_{rms} = \frac{E_{rms}}{Z}$ to recast eqn 4 in a useful equivalent way. We write

$$P_{avg} = \frac{E_{rms} I_{rms} R}{Z} = E_{rms} I_{rms} \frac{R}{Z} \quad \text{--- (7)}$$

From;



From the fig (4), $I = \frac{E_m}{Z}$

We see that R/Z is just the cosine of the phase constant ϕ :

$$\cos \phi = \frac{V_R}{E_m} = \frac{IR}{IZ} = \frac{R}{Z} \quad \text{--- (8)}$$

Eqn 7 then becomes

$$P_{avg} = E_{rms} I_{rms} \cos \phi \quad \text{--- (9)}$$

where $\cos \phi$ is the power factor. Because $\cos \phi = \cos(-\phi)$, i.e. eqn 9 is independent of the sign of the phase constant ϕ .

To maximize the rate at which energy is supplied to a resistive load in an RLC circuit, we should keep the power factor $\cos \phi$ as close to unity as possible. This is equivalent to keeping the phase constant ϕ in eq $i = I \sin(\omega t - \phi)$ as close to zero as possible.

If, for example, we ... less so by putting more capacitance to the circuit connected in series. (Recall that putting an additional capacitance into a series of capacitances decreases the equivalent capacitance C_{eq} of the series.) Thus, the resulting decrease in C_{eq} in the circuit reduces the phase constant and increases the power factor in eqn 9. Power companies place series-connected capacitors throughout their transmission systems to get these results.

(b) in the NOTE (1)

Example - 1.

A series RLC circuit, driven with $\mathcal{E}_{rms} = 120V$ at frequency $f = 60.0Hz$ contains a reactance $R = 200\Omega$, an inductance with $X_L = 80.0\Omega$ and a capacitance with $X_C = 150\Omega$.

(a) What are the power factor $\cos \phi$ and phase constant ϕ of the circuit?

Sol. NOTE: The power factor $\cos \phi$ can be found from the resistance R and impedance Z via eqn 8 ($\cos \phi = R/Z$).

To calculate Z , we use $Z = \sqrt{R^2 + (X_L - X_C)^2}$ [series connection]

$$Z = \sqrt{(200)^2 + (80.0 - 150.0)^2} = 211.90\Omega$$

$$\text{From 8, } \cos \phi = \frac{R}{Z} = \frac{200\Omega}{211.90\Omega} = 0.9438 \approx \underline{\underline{0.944}} \text{ (power factor)}$$

$$\cos \phi = 0.944$$

Taking the inverse of cosine to get the phase constant

$$\phi = \cos^{-1} 0.944 = \underline{\underline{\pm 19.3^\circ}}$$

Both $+19.3^\circ$ and -19.3° have a cosine of 0.944. To determine which sign is correct, we must consider whether the current leads or lags the driving emf. Because $X_C > X_L$, this circuit is mainly capacitive, with the current leading the emf. Thus, ϕ must be negative

$$\therefore \phi = -19.3^\circ$$

We would instead, have found ϕ with $\tan \phi = \frac{X_L - X_C}{R}$. A calculator would have give us the answer with the minus sign.

(3)

(b) What is the average rate P_{avg} at which energy is dissipated in the resistor? ^{sub}

NOTE: There are two ways and two ideas to use:
 (1) Because the circuit is assumed to be in steady-state operation, the rate at which energy is dissipated in the resistor is equal to the rate at which energy is applied to the circuit as given by eqn 9 ($P_{avg} = E_{rms} I_{rms} \cos \phi$).

(2) The rate at which energy is dissipated in a resistor R depends on the square of the rms current I_{rms} through it, according to eqn 4 ($P_{avg} = I_{rms}^2 R$).

First way: We are given the rms driving emf E_{rms} and we already know $\cos \phi$ from (a). The rms current I_{rms} is determined by the rms value of the driving emf and the circuit's impedance Z (which we know), according to eqn 6.

$$I_{rms} = \frac{E_{rms}}{Z}$$

Put I_{rms} into eqn 9, then,

$$P_{avg} = E_{rms} I_{rms} \cos \phi = \frac{E_{rms}^2}{Z} \cos \phi$$

$$= \frac{(120)^2 (0.9438)}{211.90} = \underline{\underline{64.1W}}$$

or

Second way:

$$P_{avg} = I_{rms}^2 R = \frac{E_{rms}^2 R}{Z^2} = \frac{(120)^2 (200)}{(211.90)^2} = \underline{\underline{64.1W}}$$

(c) What new capacitance C_{new} is needed to maximize P_{avg} if the other parameters of the circuit are not changed?

NOTE (1) The average rate P_{avg} at which energy is supplied and dissipated is maximized if the circuit is brought into resonance with the driving emf.

(ii) Resonance occurs when $X_C = X_L$

Calculations: From the given data, we have $X_C > X_L$. To reach resonance, we must decrease X_C .

From $X_C = \frac{1}{\omega C}$, we see that this means we must increase C to the new value C_{new} .

Using $X_C = \frac{1}{\omega C}$, we can write the resonance condition

$$X_C = X_L \quad \text{as} \quad \frac{1}{\omega C_{new}} = X_L$$

(4)

$$\frac{1}{W C_{new}} = X_L$$

but $W = 2\pi f$ (because we are given f and not ω) and the
solving for C_{new} , we find

$$C_{new} = \frac{1}{2\pi f X_L} = \frac{1}{(2\pi) \times 60 \times (80.0)}$$
$$= 3.32 \times 10^{-5} F = 33.2 \mu F.$$

Following the procedure of part (b), you can show that with
 C_{new} , P_{avg} would then be at its maximum value of 72.0 W.

(5)

PHY 152: Electricity and
Magnetism

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Course sub-outline

- ▶ Electric current and conductivity
- ▶ Types of material
- ▶ Properties of conductors
- ▶ Semiconductors
- ▶ Application of semiconductor-Diode

Suggested textbooks and references

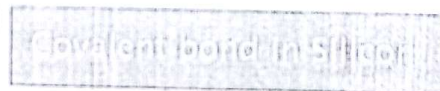
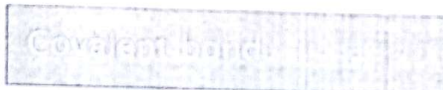
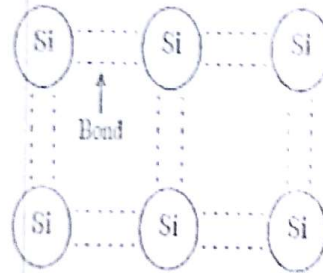
1. "Solid State Electronic Devices", Streetman and Banerjee, 5th ed., Prentice-Hall Intl Editions, ISBN 0-13-025538-6, 2000.
2. Physics for Scientists and Engineers by Fishbane et al., 2nd edition. Publisher: Prentice Hall, Upper Saddle River, New Jersey 07458
3. International Edition Physics, 5th Edition by Giancoli
4. Electronic Devices and Circuit by J.B. Gupta. Katson Educational Series.
5. Lecture notes on Introduction to diodes by F. Najimabadi, ECE65_W12_pdf
- ▶ 6. Lecture note EE2-Semiconductor device
k.fobelets@imperial.ac.uk

Objectives

- ▶ At the end of this programme, *students should be able to* :
1. Classify materials according to their electrical conductivity.
 2. Define and apply terms that are applicable to conduction of electricity such as conductivity, mobility, resistivity etc.
 3. Discuss the properties of semiconductors as distinct from that of conductors
 4. Appreciate the application and importance of semiconductor devices in the field of electronics.

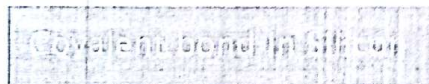
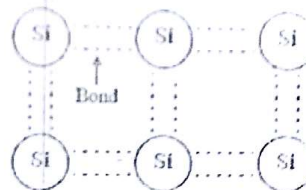
Covalent bond

- ▶ Sometimes valence electrons are shared, becoming a bond between two atoms - covalent bonding.
- ▶ This is the bonding type in diamond-crystal lattice semiconductors such as silicon semiconductors.
- ▶ An almost continuous band of allowed energies of electrons comes about when atoms are brought in close proximity to each other, this is because of the inter-atomic forces coupled with the prediction of Pauli exclusion principle.

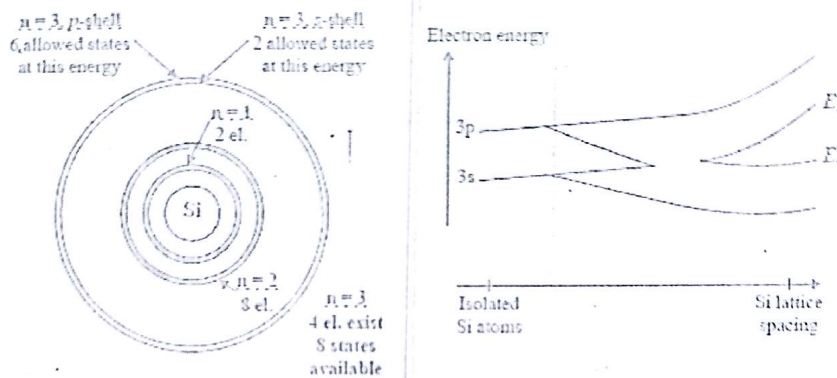


Covalent bond

- ▶ According to Pauli exclusion principle at most only two electrons can occupy an energy level at the same time.
- ▶ So one energy level is split into N levels when N atoms are brought together, and these N levels can accommodate at most 2N electrons due to spin degeneracy.



Covalent bond and Energy band

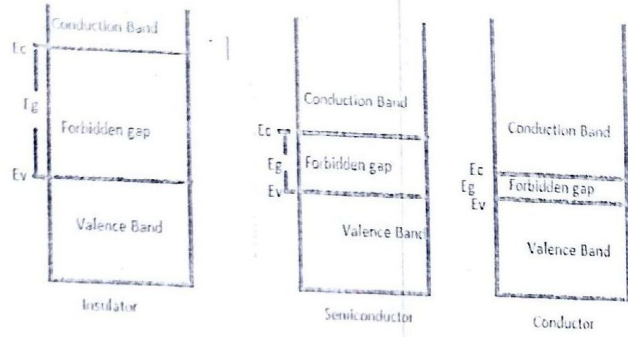


(left) Isolated Si atom, having 14 electrons. (right) Energy bands are forming when a huge number of atoms are brought together.

Energy band

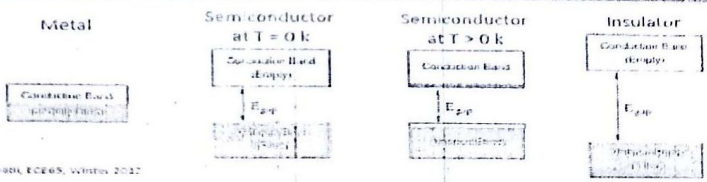
- ▶ Now, since the separation between the energy levels within the band is much smaller than the thermal energy possessed by an electron *at room temperature the band can be viewed as continuous.*
- ▶ E_c is the lowest possible conduction band energy, while E_v is the highest possible valence band energy.
- ▶ The band gap energy, E_g , is furthermore defined as $(E_c - E_v)$.
- ▶ E_g is the energy it takes to break a bond in the spatial view of the crystal. It is the factor that determine whether a material is or not a conductor at ordinary room temperatures. The lower the energy gap the more readily the material conducts.
- ▶ The band gap energies for some semiconductors at $T=300\text{ K}$ are:
 - $E_g = 1.42\text{ eV}$ in Galanium- Ascenide (GaAs) and 1.12 eV in Silicon (Si).
- ▶ For insulators: $E_g \sim 8\text{ eV}$ (SiO_2) and $\sim 5\text{ eV}$ (diamond)
- ▶ (where $1\text{ eV} = 1.602 \times 10^{-19}\text{ J}$)

Material and electrical conduction



Difference between conductors, semiconductors and insulators

- In a metal, the conduction band is partially filled. These electrons can move easily in the material and conduct heat and electricity (Conductors).
- In a semiconductor at 0 K the conduction band is empty and valence band is full. The band-gap is small enough that at room temperature some electrons move to the conduction band and material conducts electricity.
- An insulator is similar to a semiconductor but with a larger band-gap. Thus, at room temperature very few electrons are in the conduction band.



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Properties of conductors (1): Current density

▶ Electric current, i the total charge that passes through a unit cross-sectional area per unit time.

$$i = dq/dt \tag{1}$$

▶ Unit of current is Ampere, A

▶ The charge, q passing through a plane in a time interval between 0 and t is defined as

$$q = \int dq = \int i dt \tag{2}$$

▶ Unit of charge is Coulomb, C

▶ Current density, J is the rate of flow of charge per unit area through an infinitesimal area of a conductor:

$$\begin{aligned} di/dA &= J \\ di &= J dA \cos\theta \end{aligned} \tag{3}$$

Properties of conductors (2)

▶ Equation (3) shows that current density is a vector, putting the magnitude and direction of charge flow into consideration.

▶ Integrating equation (3) gives another definition of current:

$$\begin{aligned} I &= \int J \cdot dA \\ &= J \cdot A \end{aligned} \tag{4}$$

▶ Current density is then defined as: the total current flowing through a conduct per unit cross-sectional area:

$$J = I/A \tag{5}$$

Properties of conductors (3): Charge Density

- Consider charge $q = ne$ flowing through a conductor of finite area A at a velocity v , the current through the area is given as:

$$i = dq/dt \\ = nevA$$

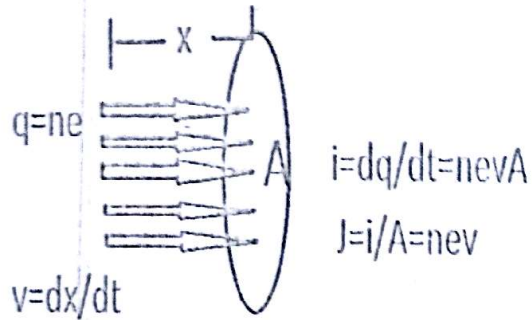
- The current density then can be expressed as:

$$J = i/A = nev \quad (6)$$

Or

$$J = \rho v \quad (7)$$

Where we have defined $\rho = ne$,
the charge density (unit = Coulomb/m³)



$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$ (Coulomb's Law)

Number of electrons flowing through a conductor is given as $n = \frac{q}{e}$

Properties of conductors (4): Drift Velocity

- The electrical force, F on the charge $q = e$ flowing through a conductor is given as

$$F = eE \quad (8)$$

here E , is the electric field, e is electron charge and n is number of electrons.

- This force must be equal to the force,

$$F = ma \quad (9)$$

producing the motion of charge, thus:

$$ma = eE \\ a = eE/m \quad (10)$$

Where a is the acceleration of the charge.

$$a = v/T \\ = eE/m \quad \text{or} \\ v = (eE/m)T \quad (11)$$

Where v is known as the Drift velocity of charges and T is the relaxation time.

Properties of conductors(5): Electrical Conductivity

- ▶ Applying equation (11) in (6) and (7):

$$\begin{aligned} J &= env \\ &= en(eE/m)T \\ &= \rho(eE/m)T \end{aligned} \quad (6)$$

- ▶ Leading to:

$$J = \rho \frac{eE}{m} T = \frac{ne^2 E}{m} T \quad (12)$$

- ▶ We reduce the equation to

$$J = \sigma E \quad (13)$$

- ▶ Here

$$\sigma = \frac{ne^2 \tau}{m} \quad (14)$$

a constant known as the *electrical conductivity of the material*.

Properties of conductors(6): Mobility & Resistivity

- ▶ From equation (14), we define the term

$$\mu_e = \frac{e\tau}{m} \quad (15)$$

known as the mobility of electrons due to the presence of electric field, E.

- ▶ **Mobility of electrons in metallic conductor is defined as the steady state drift velocity per unit electric field.**

- ▶ Recall equation (13):

$$J = \sigma E$$

- ▶ We define the term resistivity ρ (not to be mistaken with charge density) as

$$\rho = 1/\sigma \quad (16)$$

- ▶ Giving another expression for the current density:

$$J = (1/\rho)E \quad (17)$$

Properties of conductors (7): Resistivity

- Where the electric field intensity,
 $E = V/L$
and current,

$$\begin{aligned} I &= JA \\ &= (1/\rho)AE \\ &= (1/\rho)(V/L)A \end{aligned} \quad (18)$$

- Recall Ohm's law:

$$\begin{aligned} V &= IR \\ &= (VA/\rho L)R \end{aligned}$$

- Leading to:

$$\rho = (A/L)R \quad (19)$$

Where ρ is known as the resistivity of the material.

Properties of conductors (8): Resistivity

- Thus for a conductor of unit cross sectional area and of unit length, the resistivity is independent of the dimension but is dependent on the material of which it is made. The unit is Ωm .
- However, the resistivity of some materials depend on temperature; for instance the resistivity of copper varies directly with temperature to about $8500C$. For such materials we can express the temperature dependence of resistivity as

$$\rho = \rho_0 [1 + \alpha(T - T_0)] \quad (20)$$

- Since the resistance of any material is proportional to its resistivity, resistance also varies with temperature and this variation may be expressed as

$$R = R_0 [1 + \alpha(T - T_0)] \quad (21)$$

- Where R and R_0 are the resistance at temp. T and T_0 respectively; α is the temperature coefficient of resistivity.

Worked example:1

- ▶ A copper conductor of square cross section 1 mm on a side carries a constant current of 20 A. The density of free electrons is 8×10^{28} electrons per cubic meter. Find the current density and the drift velocity.

▶ **Solution:**

The current density in the wire is

$$J = \frac{I}{A} = 20 \times 10^6 \text{ A} \cdot \text{m}^{-2}$$

- ▶ From equation (6):

$$v = \frac{J}{nq} = \frac{(20 \times 10^6)}{(8 \times 10^{28})(1.6 \times 10^{-19})} = 1 \times 10^{-3} \text{ ms}^{-1}$$

Example:2

Calculate the drift velocity of the free electrons in a copper wire of cross sectional area 1.0 mm^2 when the current flowing through the wire is 2.0 A. (Number of free electrons in copper is $1 \times 10^{29} \text{ m}^{-3}$).

Solution:

Using equation 6: $I = envA$

$$\begin{aligned} \text{Drift velocity } v &= \frac{I}{enA} \\ &= \frac{2.0}{(1.0 \times 10^{29})(1.0 \times 10^{-6})(1.6 \times 10^{-19})} \\ &= 1.25 \times 10^{-4} \text{ ms}^{-1} \end{aligned}$$

Example:3

The following data is known for a conductor: Fermi energy = 5.5 eV, Mobility of electrons = $7.04 \times 10^{-3} \text{ m}^2/\text{V}\cdot\text{s}$, Number of electrons/ $\text{m}^3 = 5.8 \times 10^{28}$. Calculate (i) Relaxation time (ii) Resistivity of conductor and (iii) velocity of electrons with the Fermi energy ($e = 1.6 \times 10^{-19} \text{ C}$, $m = 9.1 \times 10^{-31} \text{ kg}$).

Solution.

- (i) Using equation (15), relaxation time is given as

$$\tau = \frac{\mu_e}{\sigma} \times m = \frac{7.04 \times 10^{-3} \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}} = 40.04 \times 10^{-15} \text{ sec.}$$

- (ii) Resistivity of conductor, using equation (17) and (14):

$$\rho = \frac{1}{\sigma} = \frac{1}{ne\mu_e} = \frac{1}{5.8 \times 10^{28} \times 1.6 \times 10^{-19} \times 7.04 \times 10^{-3}} = 1.531 \times 10^{-9} \Omega\text{m}$$

- (iii) Velocity of electrons with Fermi energy. The velocity v_F of an electron with Fermi energy E_F is given as $\frac{1}{2}mv_F^2 = E_F$, hence

$$v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2 \times 5.5 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 1.3907 \times 10^6 \text{ m/s}$$

Example:4

- ▶ A copper conductor of square cross-section 1 mm on a side carries a constant current of 20 A. Suppose the resistance is 1.72Ω at a temperature of 20°C . Find the resistance at 0°C and at 100°C (Temperature coefficients of resistivity for copper is $0.00393 / ^\circ\text{C}$).

- ▶ **Solution:**

- ▶ Using equation (21) with $T_0 = 20^\circ\text{C}$ and $R_0 = 1.72 \Omega$. Thus at $T = 0^\circ\text{C}$,

$$R = 1.72[1 + (0.00393)(0 - 20)] = 1.58 \Omega$$

- ▶ And at $T = 100^\circ\text{C}$

$$R = 1.72[1 + (0.00393)(100 - 20)] = 2.26 \Omega$$

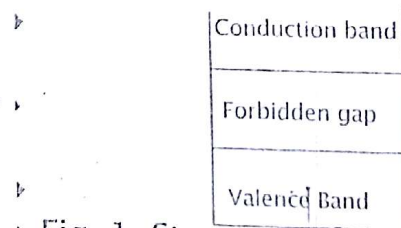
Semiconductors

»» See lecture 2 (page 7) on the PDF note

Semiconductors

- ▶ **Definition:** Semiconductors are materials which will not conduct electricity at room temperatures but become conductive when their temperatures are increased.
- ▶ In semiconductors, the highest - energy electrons fill a band (the valent band) at temperature $T=0$, as in insulators. However, unlike insulators, semiconductors have a small energy gap between that band and the next, the conduction band.

- ▶ Because the energy gap is so small, a modest electric field (or finite temperature) will allow some electrons to jump the gap thereby conduct electricity (see Fig. 1).



▶ Fig. 1: Structure of the semiconductor

Properties of Semiconductor

- ▶ **Resistivity:** lies in a wide range from 10^{-4} to about $0.5 \Omega\text{m}$
- ▶ **Resistance:** their resistance depends largely on various factors and therefore it can be controlled. The resistance decreases with increase in temperature in a manner that temperature coefficient of resistance is negative; so that a semiconductor behaves like insulators at very low temperatures but act as conductors at high temperatures
- ▶ **Illumination:** resistivity decreases in brighter (the "light" resistivity of a semiconductor is smaller than its "dark" resistivity).
- ▶ **Electric field:** resistivity depends on the magnitude of the electric field in the semiconductor, so that the current in it is not proportional to the voltage, but increases far more than the voltage i.e. semiconductors are non-linear resistors.
- ▶ **Impurities:** resistivity of semiconductors changes considerably when even minute amounts of certain other substance, called impurities, are added to them.

Types of semiconductors

- ▶ Semiconductors can be grouped into two broad categories:
 1. Intrinsic Semiconductors and
 2. Extrinsic semiconductors.
- ▶ An intrinsic semiconductor is one which is made of semiconductor material in its extremely pure form. Such semiconductors have impurity content that is less than one part in 10 billion parts of semiconductor i.e. the impurity level is very low. There are many semiconductor materials such as germanium, silicon, grey crystalline tin, selenium, tellurium, boron e.t.c. but silicon (Si) and germanium (Ge) are the two most widely used semiconductor materials in electronics. This is because
 - ▶ (i) the energy required to break their covalent bonds is very small (1.12 eV for silicon and 0.72 eV for germanium)
 - ▶ (ii) both elements have the same crystal structure and similar characteristics

Types of semiconductors

- ▶ Extrinsic semiconductors
- ▶ Intrinsic (pure) semiconductor by itself is of little significance as it has little current conduction capability at ordinary temperatures. However, the electrical conductivity of intrinsic semiconductor can be increased many times by adding very small amount of impurity to it in the process of crystallization. This process is called doping and the doped material is called the impurity or extrinsic semiconductor.
- ▶ Germanium and silicon are tetravalent. So the impurity or doping material may be either pentavalent or trivalent. Accordingly the impurity introduced may be of two types, either (i) donor or N-type or (ii) acceptor or P-type. Depending on the type of impurity added, the extrinsic semiconductor can be divided into two classes namely (i) N-type and (ii) P-type semiconductors.

Conduction in Semiconductor

- ▶ Conduction of electric current in semiconductor is made possible by the presence of electrons and holes.
- ▶ For intrinsic semiconductors, this occurs by any of the following: (i) raising their temperature above room temperatures (ii) exposing them to light photons and (iii) bombarding with ionising radiations.
- ▶ Any of the three methods ensures the covalent bonds holding atoms together are broken and electrons are raised to the conduction band

- ▶ At $T > 0$ k, some electrons are promoted to the conduction bands.
- ▶ A current flows when electrons in the conduction band move across the material (e.g., due to an applied electric field).
- ▶ A current also flows when electrons in the valance band jump between available slots in the valance bands (or "holes").
 - An electron moving to the left = a hole moving to the right!
 - We call this is a "hole" current to differentiate this current from that due to conduction band electrons.

Electrons in the conduction band

Currents in the material due to electrons in the conduction band

Conduction in Semiconductor

- ▶ For extrinsic semiconductors, electrons and holes are generated by means of doping; a process whereby an impurity is added to the semiconductor.
- ▶ Germanium and Silicon are tetravalent, hence the doping material can either be a trivalent or pentavalent material

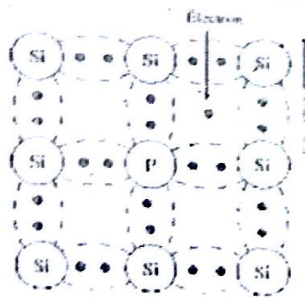
- ▶ The result is that there can be two types of extrinsic semiconductors: (i) N-type in which the majority carrier are electrons and (ii) P-type in which the majority carriers are holes

Electrons in the conduction band

Currents in the material due to electrons in the conduction band

Semiconductor: Doping & effect

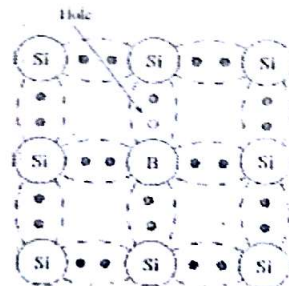
Doped n-type Semiconductor



- Donor atom (P doping) has an extra electron which is in the conduction band.
- Charge Carriers:
 - Electrons due to donor atoms
 - Electron-hole pairs due to thermal excitation
 - e: majority carrier, h: minority carrier

N-type

Doped p-type Semiconductor



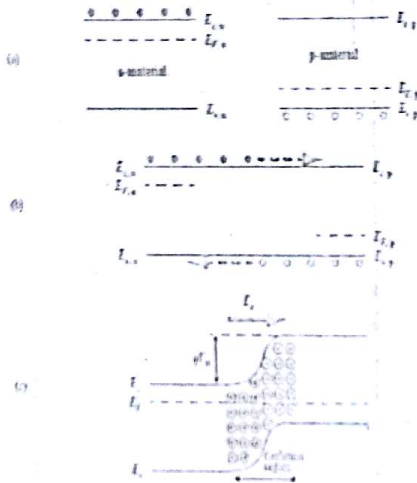
- Acceptor atom (B doping) has one less electron: a hole in the valence band.
- Charge Carriers:
 - Holes due to acceptor atoms
 - Electron-hole pairs due to thermal excitation
 - h: majority carrier, e: minority carrier

P-type

Semiconductor devices : P-N Junction (1).

- ▶ Most semiconductor devices employ one or more P-N junctions.
- ▶ **P-N junction is the control element for the performance of all semiconductor devices such as rectifiers, amplifiers, switching devices, linear and digital integrated circuits.**
- ▶ P-N junction is produced by placing a layer of P-type semiconductor next to the layer of N-type semiconductor.
- ▶ The interface separating the N and P regions is referred to as the **metallurgical junction**.

P-N junction (2)



- ▶ When the two pieces are put together, in a so-called step junction, diffusion of carriers will immediately start taking place, as depicted in Fig.4 (b).
- ▶ Electrons diffuse from the n-material to the p-material in the conduction band, while holes diffuse from the p-material to the n-material in the valence band.
- ▶ Uncompensated dopant ions are left behind on each side and the electric field that is starting to build up, because of these ions, will soon balance the diffusion.
- ▶ Fig.4(c) shows the final stage in which a balance between drift and diffusion has been established.

Fig. 4: Different stages involved in p-n

P-N junction (3)

- ▶ The electric field E_x is forcing electrons to the left and holes to the right, which exactly balances the electron diffusion to the right and the hole diffusion to the left.
- ▶ We now have a p-n junction with a built-in potential which is called the contact or barrier potential, V_0 .
- ▶ The region around the junction is called the depletion region and according to the depletion approximation it completely lacks free carriers (which is almost true).
- ▶ Fig. 5 shows the doping profile of an ideal uniformly doped P-N junction.

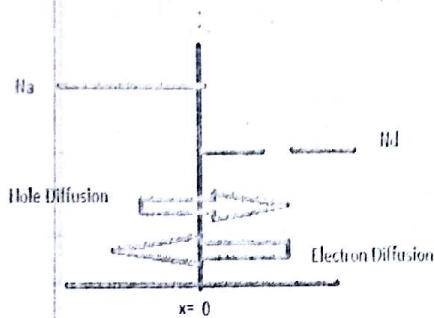
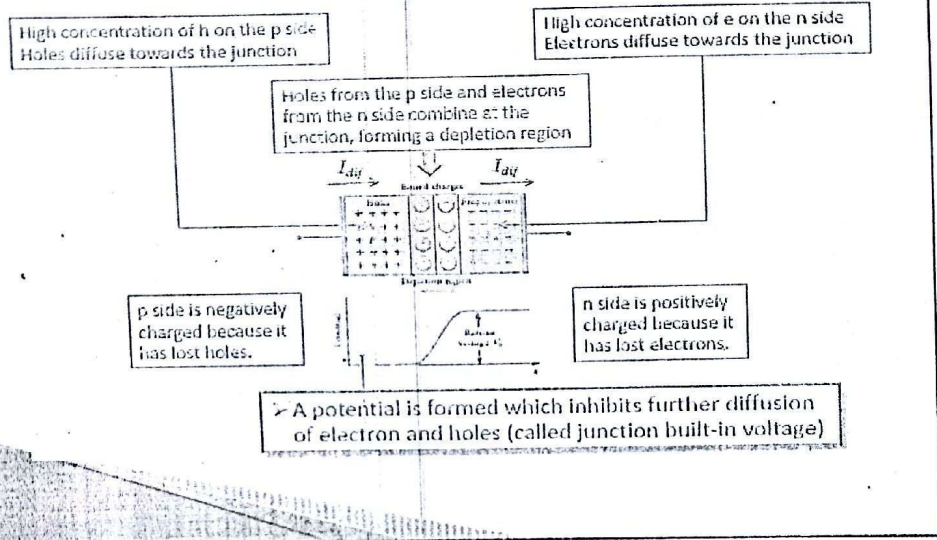


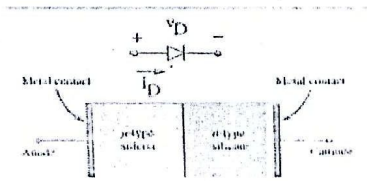
Fig. 5: Doping profile of an ideal uniformly doped P-N junction.

P-N junction (4)



P-N junction diode: Application (1)

- ▶ P-N junction diode is a device which conducts when forward biased and practically does not conduct when reverse biased.
- ▶ When an external field, with P-region connected to positive terminal and N-region connected to negative terminal of the battery is applied across the junction, the junction is said to be forward biased.
- ▶ In this circuit arrangement, the holes on the P-side being positively charged particles are repelled from the positive bias terminal and driven toward the junction. Similarly, the electrons on the N-side are repelled from the negative bias terminal and driven towards the junction.
- ▶ The result is that the depletion region is reduced in width and the barrier potential is also reduced.



Simplified physical structure

Application of diodes: Rectifier

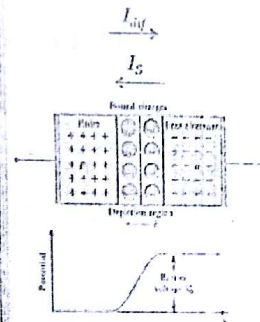
Diode in forward bias

P-N junction diode (2)

- ▶ If the applied bias voltage is increased from zero, the barrier potential gets progressively smaller until it effectively disappears and charge carriers can easily flow across the junction.
- ▶ Electrons from the N-side are then attracted to the positive bias terminal while holes from the P-side flows across to the negative bias terminal on the N-side.
- ▶ This leads to the flow of a current of majority carrier.
- ▶ Since barrier potential is very small (0.3 V for Ge and 0.7 V for Si), a small forward voltage is sufficient to eliminate the barrier completely.
- ▶ Once the barrier is eliminated, junction resistance becomes almost zero and a low resistance path is established in the entire circuit.
- ▶ The current, called forward current flows in the circuit.

P-N Junction diode (3)

- ▶ Thermally generated minority carriers on the n side (holes) move toward the depletion region, and are swept into the p side by the potential where they combine with electrons. (similar process for minority carriers on the p side). This sets up a drift current, I_S .
- ▶ To preserve charge neutrality, a non-zero $I_{diff} = I_S$ should flow (height of potential is slightly lower).
- ▶ I_{diff} scales exponentially with changes in the voltage barrier.
- ▶ I_S is independent of the voltage barrier but is a sensitive function of temperature.



Diode i-v characteristic (1)

$$i_D = I_S (e^{v_D/nV_T} - 1)$$

I_S : Reverse Saturation Current
(10^{-9} to 10^{-12} A)

V_T : Volt-equivalent temperature
(= 26 mV at room temperature)

n : Emission coefficient
($1 \leq n \leq 2$ for Si ICs)

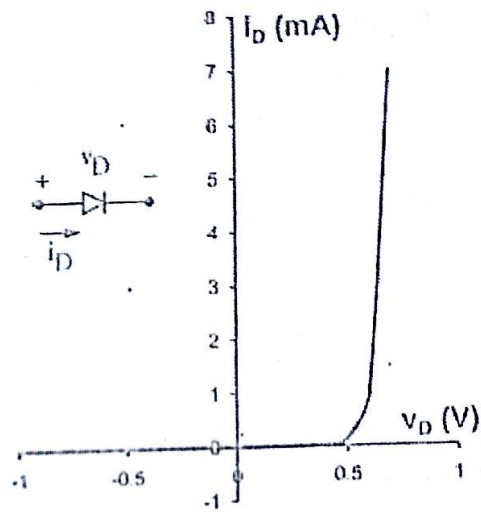
For $|v_D| \geq 3nV_T$

Forward bias: $i_D \approx I_S e^{v_D/nV_T}$

Reverse bias: $i_D \approx -I_S$

Sensitive to temperature:

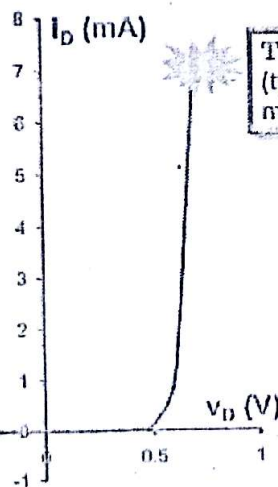
- > I_S doubles for every 7°C increase
- > $V_T = T/11,600$



Diode limitation

Reverse Breakdown at Zener voltage (V_Z)
(due to Zener or avalanche effects)

Zener diodes are made specially to operate in this region!



Thermal load, $P = i_D v_D$
(typically specified as maximum i_D)

P-N junction diode and rectification.

- ▷ The ideal diode has the property of being unidirectional in the sense that a voltage applied with given polarity will cause flow of current with a negligible resistance while a voltage of opposite polarity will give no (or negligible) current.
- ▷ As a result diodes are made to meet specific applications such as rectifiers in electronic circuits.
- ▷ As rectifiers, p-n junction diodes render current in one direction in electronic circuits.

Solved examples

1. A 100 ohm resistor is to be made at room temperature in a rectangular silicon bar of 1 cm in length and 1 mm² in cross-sectional area by doping it appropriately with phosphorous atoms. If the electron mobility in silicon at room temperature be 1,350 cm²/V-second, calculate the dopant density needed to achieve this. Neglect the insignificant contribution to the intrinsic carriers.

Solution.

Length of silicon bar, $l = 0.01 \text{ m}$

Area of cross section of silicon bar, $A = 1 \text{ mm}^2 = 1 \times 10^{-6} \text{ m}^2$

Resistance of silicon bar, $R = 100 \Omega$

Resistivity of silicon, $\rho = \frac{RA}{l} = \frac{100 \times 1 \times 10^{-6}}{0.01} = 0.01 \Omega - m$

$$\begin{aligned} \text{Dopant density, } n &= \frac{\sigma}{e\mu_n} = \frac{1}{\rho e\mu_n} \\ &= \frac{1}{0.01 \times 1.6 \times 10^{-19} \times 1.350 \times 10^{-4}} \\ &= 0.0463 \times 10^{23} \text{ m}^{-3} \\ &= 4.63 \times 10^{21} \text{ cm}^{-3} \end{aligned}$$

Example 2

2. What is the concentration of holes in Si crystals having donor concentration of $1.4 \times 10^{22}/\text{m}^3$ when the intrinsic carrier concentration is $1.4 \times 10^{15}/\text{m}^3$? Find the ratio of electrons to holes concentration.

Solution:

Intrinsic carrier concentration, $n_i = 1.4 \times 10^{15}/\text{m}^3$

Donor concentration, $N_D = 1.4 \times 10^{22}/\text{m}^3$

Concentration of electron, $n = N_D = 1.4 \times 10^{22}/\text{m}^3$

Concentration of holes, $p = \frac{n_i^2}{n} = \frac{(1.4 \times 10^{15})^2}{1.4 \times 10^{22}} = 1.4 \times 10^{12}/\text{m}^3$

Ratio of electron to hole concentration = $\frac{n}{p} = \frac{1.4 \times 10^{22}}{1.4 \times 10^{12}} = 1 \times 10^{12}$

Appendix 1

Note: For extrinsic semiconductor:

$$N_D + p = N_A + n$$

Where N_D is the concentration of donor atoms and N_A is the concentration of acceptor

In an N-type semiconductor there is no acceptor doping i.e. $N_A = 0$, also the number of electrons is much greater than the number of holes i.e. $n \gg p$.

Therefore, the concentration of holes in N-type semiconductor will be given by the equation

$$p = \frac{n_i^2}{n} = \frac{n_i^2}{N_D}$$

Similarly in case of p-type semiconductors

$$p \approx N_A$$

$$\text{And } n = \frac{n_i^2}{N_A}$$