PHY 142   PHY 142 [Atomic & Nuclear Physics]   1

Course Outline

Theory of atomic structure, Thompson, Rutherford and Bohr's atomic theories, the hydrogen atom, properties of electron, CRO, e/m, Millikan's experiment, properties of the nucleus, natural radioactivity, wave particle duality of light, X-rays, photoelectricity, Thermionic emission, diode valves.

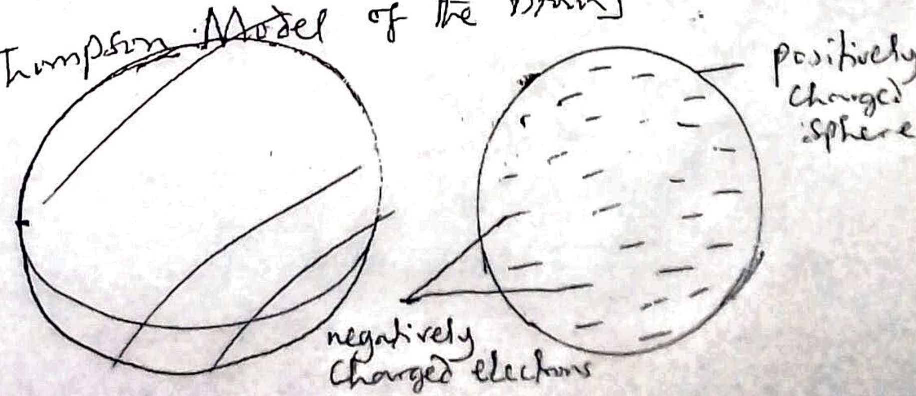
Theory of Atomic Structure

* Atoms are generally electrically neutral which implies that they must contain +ve charges equal in magnitude to the -ve charges carried by normal compliments of electrons. Thus a neutral atom has a negative charge of magnitude ZE, where E is the electronic charge and also a positive charge of the same magnitude.
* The fact that the mass of an electron is very small compared to the mass of even the smallest atom, implies that most of the mass of the atom must be associated with the +ve charge. All this considerations led to the question of distribution of +ve and -ve charges within the atom.

[Thompson's Model of the Atom]

* Thompson proposed a tentative description/model of the composition of an atom, whereby the negatively charged electrons were located within a continuous distribution of positive charges. The positive charges distribution was assumed to be spherical in shape with a radius of the order of 10^{-8} cm which is known to be the order of magnitude of the radius of an atom.
* However because of their mutual repulsion, the electrons would be uniformly distributed through the sphere of positive charges.

[Illustration of Thompson Model of the Atom]



In an atom in its lowest possible energy state, the electrons would be fixed about their equilibrium positions. In excited atoms, the electrons would vibrate about their equilibrium position. Since the electromagnetic theory predicts that an accelerated charged body such as a vibrating electron emits electromagnetic radiation, it was possible to understand qualitatively the emission of such radiation by excited atoms on the basis of Thompson model.

[Limitations of the Model]

The calculation of ~~rand~~ spectrum of radiation which would be emitted shows that the model did not appear to be able to lead to quantitative agreement with experiments. The proof of the inadequacies of thompson model was obtained by rutherford from the analysis of certain experiments involving the scattering of alpha particles by atoms.

[Rutherfords Model of the Atom]

Rutherfords observed that the passage of alpha particles (+vely charged particles) through a very thin metal foil was accompanied by some scattering of the particles from their original direction. A few of the particles suffered deflection of more than 90°. In effect they are reflected towards the source. To explain these results rutherford suggested that all the positive charges and nearly all the mass were concentrated in a very small volume and Nucleus at the Centre of the atom.

The large angle scattering of alpha particles would then be explained by the strong electrostatic repulsion to which the alpha particles are subjected on approaching closely enough to the tiny nucleus. It is believed that protons are responsible for the positive charges in the nucleus.

⇒ Rutheford considered the electrons to be outside the nucleus and at relatively large distances from it so that their negative charge did not act as a shield to the positive charge when an alpha particle penetrates the atom.

⇒ The electrons were supposed to move in circular orbit around the nucleus (like the planets moving around the sun). The electrostatic attraction between the two opposite charges provides the required centripetal force for such motion.

[Limitations of the Rutherford's Model]

Rutherford's model of the atom strongly supported by the evidence of the nucleus was not consistent with Classical electromagnetic theory which states that ~~at~~ electron moving in a circular orbit around a nucleus is accelerating and accordingly emits radiation continuously and thereby loses energy. If this happens, the radius of the orbit would progressively decrease and the electrons would spiral into the nucleus.
Evidently either the model of the atom or the classical theory of radiation required some modification. In an effort to overcome this paradox, Bohr made some postulate

[Bohr's Model of the Atom]

Bohr's postulates are given as follows;

⇒ Electrons can move round the nucleus only in certain allowed orbit and while they are in this orbit they do not emit radiation - An electron in an orbit has a definite amount of energy. It possesses kinetic energy because of its motion and potential energy on account of the attraction of the nucleus. Each allowed orbit is therefore associated with a certain quantity of energy called energy of the orbit.

⇒ Only those orbits are allowed or permitted for which the angular momentum is an integer of $h/2\pi$ where h is the planck's constant.

⇒ An electron can jump from one orbit of energy E_2 to another of lower energy E_1 and the energy difference is emitted as one quantum of radiation of frequency f given by

$$E_2 - E_1 = hf$$

By choosing the allowed orbit correctly, Bohr was able to explain quantitatively why particular wavelengths appear in a line spectrum of hydrogen and this provided evidence for his ideas concerning how electromagnetic radiation originates in an atom.

[Limitations of Bohr's Model]

Despite its considerable achievement, the Bohr's atom has certain shortcomings

... of the detail of the optical aspect.

(i) It could not interpret the detail of the atoms containing more than one electron.
(ii) the very arbitrary method of selecting allowed orbits has no theoretical basis.
(iii) It involves quantities such as radius of an orbit which could not be experimentally checked. However, credit is due to Bohr for linking spectroscopy and atomic structure and for introducing quantum idea into atomic theory.

[The Bohr Atom]

The angular momentum of a particle of mass M moving with tangential speed V in a circle of radius r is given as MVr

Applying Bohr's condition we obtain

$$\underbrace{mvr}_{\text{angular momentum mvr}} = \frac{nh}{2\pi} \quad \text{---} \quad (1)$$

where $n = (1, 2, 3, 4 \text{----} n)$

Consider a hydrogen atom having a single electron of charge $-e$ revolving about a single proton of charge $+e$. The electrostatic force of attraction between the charges is given by

$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{r^2} \quad \text{---} \quad (2)$$

Since this provides the centripetal force, we have

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^{\cancel{2}}} = \frac{mv^2}{\cancel{r}} \quad \text{---} \quad (3)$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0} e^2 = mv^2 r \quad \text{---} \quad (4)$$

$$v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 mr}}$$

dividing (4) by (1)

$$\frac{e^2}{4\pi\varepsilon_0} \Big/ \frac{nh}{2\pi} = \frac{mv^2 r}{mvr} \quad \text{---} \quad (5)$$

Rearranging gives

$$\frac{e^2 \times 2\pi}{4\pi\varepsilon_0 nh} = \frac{\cancel{m}v^{\cancel{2}}\cancel{r}}{\cancel{mvr}} \quad \text{---} \quad (5)$$

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$$\Rightarrow \frac{e^2}{2\varepsilon_0 nh} = v \quad ---- \quad (6)$$

However from (1) by rearranging it

$$r = \frac{nh}{2\pi m v} \quad ---- \quad (7)$$

putting equation (6) into (7) we obtain

$$r = \frac{nh}{2\pi m \left(\frac{e^2}{2\varepsilon_0 nh}\right)}$$

orbital radii in Bohr atom.

$$\Rightarrow r = \frac{n^2 h^2 \varepsilon_0}{\pi m e^2} \quad ---- \quad (8)$$

If $r_0 \equiv \dfrac{h^2 \varepsilon_0}{\pi m e^2}$

Then equation (8) becomes

$$r = n^2 r_0 \quad ---- \quad (9)$$

$\therefore r = r_0, 4r_0, 9r_0 \; ----$

r gives the permitted or allowed non radiating orbits and with values as a multiple of the orbital integer i.e $r_0, 4r_0, 9r_0, 16r_0, 25r_0$. n = quantum number. Since r_0 is made up of constant terms, the radius of the atom depends on the quantum no and this implies that the radius of the atom is quantized.

Find r_0 if given

$\varepsilon_0 = 8.854 \times 10^{-12}\ C^2 N^{-1} m^{-2}$

$h = 6.62 \times 10^{-34}\ Js$

$m = 9.109 \times 10^{-31}\ kg$

$e = 1.602 \times 10^{-19}\ C$

$r_0 = 5.29 \times 10^{-11}\ m$

$= .53 \times 10^{-10}\ m$

$= .53 \times 10^{-8}\ cm$

This is in good agreement with the ...

$$E = -2.17 \times 10^{-18}\ J$$

$$E = -13.6\ eV.$$

This agrees with the experimentally observed b.e for hydrogen

(Q) Examining the wave behaviour of an electron in orbit around a hydrogen nucleus.

The de Broglie wavelength of the electron is

$$\lambda = \frac{h}{mv}$$

and

$$v = \sqrt{\frac{e^2}{4\pi\epsilon_0 mr}}$$

$\therefore$ orbital electron wavelength

$$\lambda = \frac{h}{m} \times \frac{\sqrt{4\pi\epsilon_0 mr}}{e}$$

$$= \frac{me^4}{4\varepsilon_0^2 n^2 h^2}\left[\frac{1}{2} - 1\right]$$

$$\Rightarrow E = -\frac{1}{8}\frac{me^4}{\varepsilon_0^2 n^2 h^2} \quad \text{—} \quad (14)$$

$n = 1, 2, 3$

The total energy have a negatively sign because the reference level of potential energy is taken with the electron at an infinite distance from the nucleus.

We can ~~we~~ write (14) as

$$E = \frac{-R}{n^2} \quad \text{where} \quad \boxed{R = \frac{me^4}{8\varepsilon_0^2 h^2}}$$

1. Cal the binding energy of the hydrogen atom (the energy binding the electron to the nucleus)

The B.E is numerically equal to the energy of the lowest state. The largest -ve value is found for n = 1

$$\therefore E = \frac{-me^4}{8\varepsilon_0^2 n^2 h^2} = \frac{-me^4}{8\varepsilon_0^2 h^2}$$

$$= -\frac{(9.11\times10^{-31}\,kg)(1.6\times10^{-19}c)^4}{\quad}$$

$$\lambda = \frac{h}{e}\sqrt{\frac{4\pi\varepsilon_0 r}{m}}$$

7) Cal the electron wavelength if 5.3×10^{-11} m is the radius of the electron orbit.

$$\lambda = \frac{6.63 \times 10^{-34} \text{ J.s}}{1.6 \times 10^{-19} \text{ C}}\sqrt{\frac{(4\pi)(8.85 \times 10^{-12})(5.3 \times 10^{-11})}{9.1 \times 10^{-31}}}$$

$$\lambda = 33 \times 10^{-11} \text{ m}$$

The radius of the innermost orbit is customarily called Bohr radius of the hydrogen atom

$$r_0 = 5.292 \times 10^{-11} \text{ m}.$$

each value of n, there are corresponding values of orbit radius r_n, speed V_n, angular momentum $L_n = \frac{nh}{2\pi}$ and the total energy E_n.

The possible transitions from one electron orbit to an orbit of lower energy is as illustrated in the figure above.

Considering the transition from n_u to n_L, the energy of the emitted photon $\frac{hc}{\lambda}$ is equal to $E_{n_u} - E_{n_L}$.

$$\therefore \frac{hc}{\lambda} = \left(-\frac{me^4}{8\epsilon_0^2 n_u^2 h^2}\right) - \left(-\frac{me^4}{8\epsilon_0^2 n_L^2 h^2}\right)$$

$$\frac{hc}{\lambda} = -\frac{me^4}{8\epsilon_0^2 n_u^2 h^2} + \frac{me^4}{8\epsilon_0^2 n_L^2 h^2}$$

$$\therefore \frac{hc}{\lambda} = \frac{me^4}{8\epsilon_0^2 h^2}\left[\frac{1}{n_L^2} - \frac{1}{n_u^2}\right]$$

Energy of the Electron

The kinetic energy of the electron in any orbit is given by

$$KE = \frac{1}{2}mv^2 \quad \text{------} \quad (10)$$

putting (6) into (10)

$$KE = \frac{1}{2} m \frac{e^4}{\varepsilon_0^2 4 n^2 h^2}$$

$$KE = \frac{1}{8} \frac{me^4}{\varepsilon_0^2 n^2 h^2} \quad \text{---} \quad (11)$$

Recall that $PE = Fdr = dW$

$\therefore$ total work done $= W = \int_0^r F dr$ --- (12)

$$PE = \int_0^r \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2} dr$$

$$= \frac{e^2}{4\pi\varepsilon_0} \int_0^r \frac{dr}{r^2}$$

$$= \frac{e^2}{4\pi\varepsilon_0} \left[-\frac{1}{r} \right]$$

$$\therefore PE = \frac{-e^2}{4\pi\varepsilon_0 r} \quad \text{----} \quad (13)$$

putting (8) into (13)

$$PE = \frac{-me^4}{4\varepsilon_0^2 n^2 h^2}$$

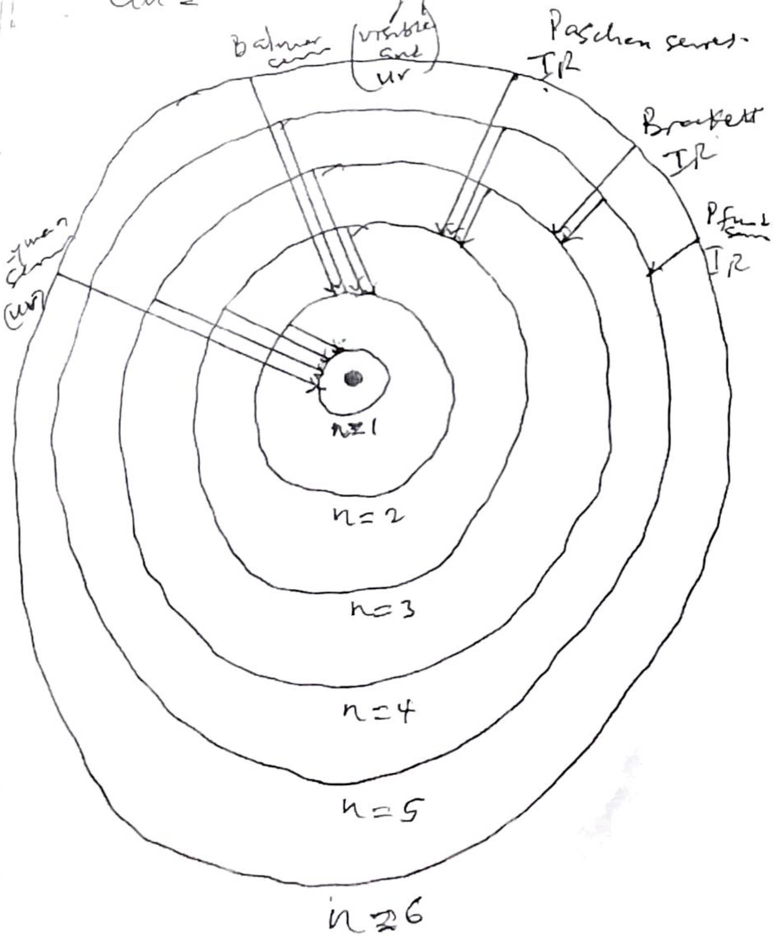
Since work ~~is~~ done/ is required to move the electron from this point to infinity against the attraction of the nucleus, the potential energy of the electron is negative.

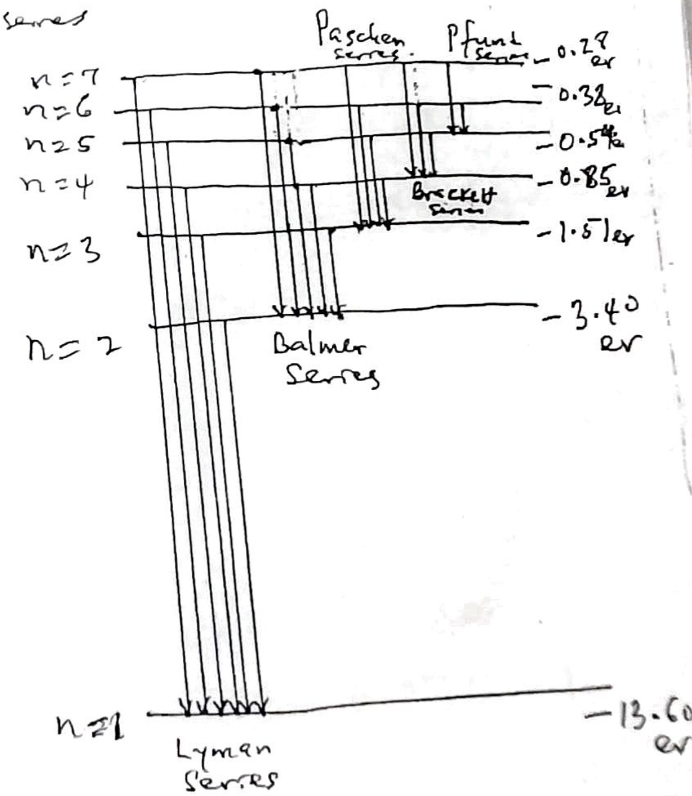
Total Energy = KE + PE

$$= \frac{1}{8} \frac{me^4}{\varepsilon_0^2 n^2 h^2} - \frac{1}{4} \frac{me^4}{\varepsilon_0^2 n^2 h^2}$$

E_n in equation (14) has a different value for each n. We can obtain the energy levels of the hydrogen atom in the Bohr model. Each distinct orbit corresponds to a distinct energy level

The figure below depicts the orbits and energy levels





These represent the energy levels of the hydrogen atom and the transition btw them

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$$\frac{1}{\lambda} = \frac{me^4}{8\varepsilon_0^2 h^3 c}\left[\frac{1}{n_L^2} - \frac{1}{n_u^2}\right]$$

$$\frac{1}{\lambda} = R\left[\frac{1}{n_L^2} - \frac{1}{n_u^2}\right]$$

hydrogen wavelength in the Bohr model $n_L < n_u$.

R = Rydberg constant

$$R = \frac{me^4}{8\varepsilon_0^2 h^3 c}$$

Substituting for m, c, e, h and ε_0

$R = 1.097 \times 10^7\ m^{-1} = 1.0967758 \times 10^{-3}\ Å^{-1}$

And $\bar{\nu} = \frac{1}{\lambda}$ where $\bar{\nu}$ is the emitted wave number.

$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$, $\varepsilon_0 = \frac{1}{4\pi \times 9 \times 10^9}$

~~$\varepsilon_0 = 4\pi \times 9 \times 10^9$~~

$\frac{1}{\varepsilon_0} = 4\pi \times 9 \times 10^9$

$$\therefore R = \frac{me^4}{8h^3c} \times (4\pi \times 9 \times 10^9)^2$$

$= 1.097 \times 10^7\ m^{-1}$

The wavelength of the first line of Balmer series of hydrogen is 6563×10^{-10} m.
Cal. the wavelength of its second line.

For Balmer series

1^{st} line $\Rightarrow$ $n_1 = 2$, $n_2 = 3$

2^{nd} line $\Rightarrow$ $n_1 = 2$, $n_2 = 4$.

$$\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36} \quad \text{——— (1)}$$

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3}{16}R \quad \text{——— (2)}$$

dividing (1) by (2)

$$\frac{\lambda_2}{\lambda_1} = \frac{20}{27} \quad \text{and} \quad \lambda_1 = 6563 \times 10^{-10}$$

$$\therefore \lambda_2 = \frac{20 \times 6563 \times 10^{-10}}{27} = 4861 \times 10^{-10}\,m$$