

COMPTON EFFECT

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Physics 302

The Compton effect or Compton scattering is the scattering of photons by free electrons. Classical electromagnetic theory says that when an electromagnetic wave of frequency ν is incident on free electrons, the electrons absorb electromagnetic radiation and oscillate with frequency ν . The oscillating electrons, in turn, reradiate electromagnetic waves of the same frequency ν . This is referred to as coherent scattering, i.e., there is no change in the wavelength.

The scattering of very short wavelength radiation is observed to disagree with this. The scattered radiation is observed to consist of two frequencies: the original frequency ν (or wavelength λ) and another frequency $\nu' < \nu$ (or wavelength $\lambda' > \lambda$).

The correct explanation of the modified wavelength in x-ray scattering was given by A. H. Compton in 1922. He adopted the quantum hypothesis and by applying the laws of conservation of momentum and energy, was able to calculate the change in frequency or wavelength of the radiation.

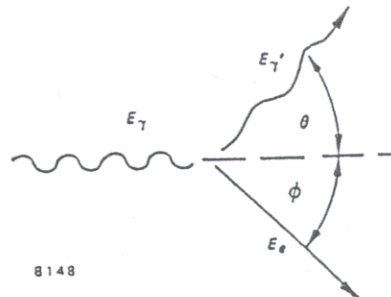


Fig. 1. Scattering Caused by Compton Interaction.

Figure 1. shows schematically the collision between the incident photon and a free electron (assumed to be at rest). The photon has energy $h\nu$ and momentum $h\nu/c$ and the electron has rest mass energy of m_0c^2 . After the collision, the scattered photon has an energy of $h\nu'$, ($< h\nu$) and momentum $h\nu'/c$ ($< h\nu/c$). The wavelength of the scattered photon λ' is larger than λ . The decrease in the energy of the photon, namely $h\nu - h\nu'$, appears as the kinetic energy K_e of the recoil electron. From relativity

$$K_e = mc^2 - m_0c^2 = m_0c^2 \left[\frac{1}{\sqrt{1 - \beta^2}} - 1 \right]$$

where $\beta = v/c$, v being the velocity of the recoil electron. The relativistic momentum of the electron, p_e is

$$p_e = mv = m\beta c = \frac{m_0\beta c}{\sqrt{1-\beta^2}}.$$

Now suppose that the scattered photon and the recoiling electron make angles θ and ϕ respectively, with the direction of the incident photon, as shown in Fig. 1. Applying the conservation of momentum to the collision for the X and Y components, yields

$$\frac{h\nu}{c} = \frac{h\nu'}{c} \cos \theta + p_e \cos \phi \quad (1)$$

$$0 = \frac{h\nu'}{c} \sin \theta - p_e \sin \phi, \quad (2)$$

The conservation of energy gives

$$h\nu = h\nu' + K_e. \quad (3)$$

The solution of the above set is found by rewriting the momentum equations as

$$p_e c \cos \phi = h\nu - h\nu' \cos \theta, \quad (4)$$

$$p_e c \sin \phi = h\nu' \sin \theta. \quad (5)$$

Squaring and adding these equations yields

$$p_e^2 c^2 = (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu') \cos \theta. \quad (6)$$

The total energy E_e of the recoiling electron is

$$E_e = K_e + m_0 c^2 = (h\nu - h\nu') + m_0 c^2, \quad (7)$$

which from relativity is also equal to

$$E_e = \sqrt{p_e^2 c^2 + m_0^2 c^4}. \quad (8)$$

Equating Eqs. (7) and (8) and squaring, yields

$$p_e^2 c^2 = (h\nu - h\nu')^2 + 2(h\nu - h\nu') m_0 c^2. \quad (9)$$

Again equating the values of $p_e^2 c^2$ from Eqs (6) and (9) and then solving for $h\nu'$, yields

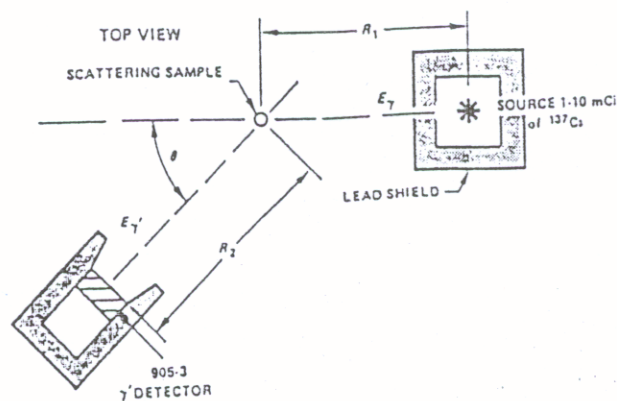
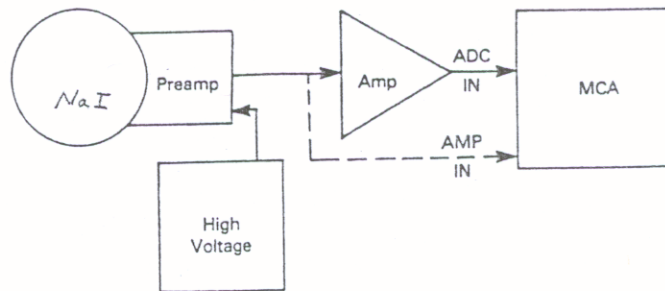
$$h\nu' = \frac{h\nu}{1 + (h\nu/m_0c^2)(1 - \cos \theta)} \quad (10)$$

By using the relation $\lambda\nu = \lambda'\nu' = c$ we may write the change in wavelength $\Delta\lambda = \lambda' - \lambda$, due to the Compton effect as

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0c}(1 - \cos \theta). \quad (11)$$

Experimental Method

The first thing to do is to assemble the electronics as shown in the Fig. 2. The NaI should be a 2 inch diameter crystal. It should have a lead shield around the crystal (lead brick with a 2 inch hole). The high voltage for the detector should be set at +1000 V. The amplifier gain should be chosen so that a ^{60}Co spectrum covers about 1000 channels. Using a ^{137}Cs , a ^{60}Co source, and a ^{22}Na source obtain a calibration curve for the multichannel analyzer. Have the lab instructor place the container with the 150 millicurie ^{137}Cs source on the table in the proper position as shown in Fig. 3. Place lead bricks so that the gamma-ray beam is collimated to the size of the aluminum scatterer. Place other lead bricks so that the detector is shielded from the source by two to three bricks. Acquire a spectrum at several angles and determine the energy of the photopeak of the Compton scattered photon using your calibration curve.



Compton Scattering - Experimental Method (addendum)

The data acquisition system has already been calibrated for this experiment. The PC displays the number of counts (number of gamma rays entering the detector) versus the energy (in keV) of the detected gamma ray (or other event). To utilize the data acquisition system you should familiarize yourself with the following operations of the system:

- (a) Acquire data.
- (b) Stop the acquisition of data.
- (c) Acquire for a preset time.
- (d) Change the vertical scale.
- (e) Set up a region of interest.
- (f) Print a graph of the display.
- (g) Save a spectrum.

Part 1

Verify the correctness of the calibration by placing ^{60}Co source in front of the detector. There should be two peaks at 1172 keV and 1332 keV. Remove the ^{60}Co source and place the ^{137}Cs source in front of the detector. There should be a peak at 662 keV. It may be necessary to correct the calibration by adding an offset. Ask the instructor about this.

Part 2

Have the lab instructor place the 150 millicurie ^{137}Cs on the table in the proper position as shown in Figure 3. Acquire a spectrum at scattering angles of 20° to 90° in increments of 10° and determine the energy of the photopeak of the Compton scattered photon.

Analysis

Plot E_γ (calculated) vs θ on linear graph paper. Put the experimental points with the estimated error on the curve. Another analysis which gives a curve that is easily fitted is obtained by writing Eq. 10 as

$$E_\gamma = \frac{E_\gamma}{1 + 1.956 E_\gamma (1 - \cos \theta)}$$

and

$$\frac{1}{E_\gamma} = 1.511 + 1.956(1 - \cos \theta).$$

A plot of $\frac{1}{E_\gamma}$ vs $(1 - \cos \theta)$ should be a straight line with intercept 1.51 and a slope equal to 1.956. Make such a plot and do a least squares fit to the data.

$\theta(\text{deg})$	$E_\gamma(\text{Calculated})$	$1E_\gamma(\text{Measured})$
20		
40		
50		
60		
70		
80		
90		