Write on both sides of the paper Question .... WK 5&6

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Magnetic Force on a Current-Carrying Wire

~~From Hall effect, we have seen that~~ a magnetic field exerts a sideways force on electrons moving in a wire (i.e ~~current~~ electrons moving or drifting in current carrying wire). Because the conduction electrons cannot escape sideways out of the wire, this force is transmitted to the wire.

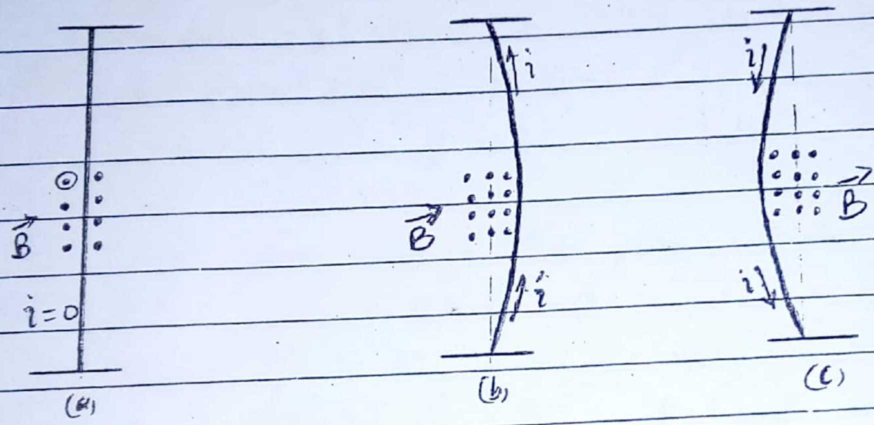

fig 1

The diagrams above show a vertical wires fixed in place at both ends, extends through the gap between the pole faces a magnet. The direction of the magnetic field is outward from the page. In (a), there is no current through the wire and there is no deflection. In (b) current is sent upward through the wire and the wire deflects to the right. In (c), current is reverse and the wire deflects to the ~~right~~ left

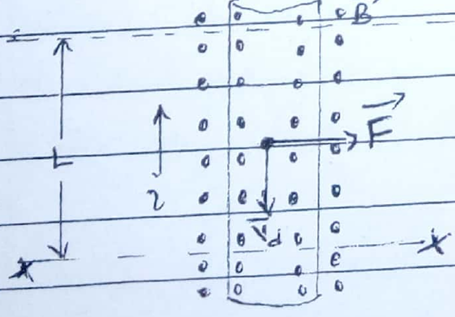

Fig 2 ~~shows ex~~ is a picture of 1(b)

fig 2

In figure 2, if either the direction of the magnetic field or the direction of the current is reversed, the force

WK 9 ϕ

on the wire would be reversed. Note that it does not matter whether we consider -ve charges drifting downward in the wire or +ve charges drifting upward, the direction of deflecting force on the wire is the same.

Let ~~us~~ consider a wire of length L in fig 2, All the conduction electrons in this section of the wire will drift past plane xx in a time $t = L/V_d$. Thus, in that time a charge

$$q = it = i\frac{L}{V_d}$$

will pass through that plane. ~~Subst~~

but $F_B = qV_dB\sin\phi$

Where ϕ is the angle between $\vec{V}_d$ and $\vec{B}$ and from the diagram (fig 2) $\phi = 90°$. Put q in F_B.

$$F_B = \frac{iL}{V_d}V_dB\sin 90°$$

$$F_B = iLB \quad \text{——} \quad (22)$$

This equation gives the magnetic force that acts on a length L of straight wire carrying a current i and immersed in a magnetic field $\vec{B}$ that is $\perp$ to the the wire.

The generalized form of eqn (22) is given as

$$\vec{F_B} = i\vec{L}\times\vec{B} \quad \text{——} \quad (23)$$

where $\vec{L}$ is a length vector that has magnitude L and is directed along the wire segment in the direction of current.

The magnitude of $\vec{F_B}$ is

$$F_B = iLB\sin\phi \quad \text{——} \quad (24)$$

where ϕ is the angle between the direction of $\vec{L}$ and $\vec{B}$.

If a wire is not straight or the field is not uniform, we can imagine it broken up into small straight segments and applying eqn (23) to each segment

Write on both sides of the paper   Question ....

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The force on the whole wire is the vector sum of all the forces on the segments that make up the wire.

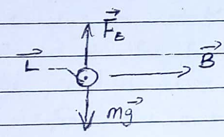
Example.

A straight, horizontal length of copper wire has a current $i = 28A$ through it. What are the magnitude and direction of the minimum magnetic field $\vec{B}$ needed to suspend the wire – that is, to balance the gravitational ~~field~~ force on it? The linear density (mass per unit length) of the wire is 46.6g/m.

Soln

$46.6L \times g = iLB$

required diagram



Q. A horizontal conductor that is part of a power line carries a current of 5000A from south to north. Earth's magnetic field (60.0μT) is directed toward the north and is inclined downward at 70° to the horizontal. Find the magnitude and direction of the magnetic force on 100m of the conductor due to Earth's field.

Q. A wire 50cm long lying along the x axis carries a current of 0.50A in +ve x direction, through a magnetic field $\vec{B} = (0.0030T)\hat{j} + (0.010T)\hat{k}$. Find the magnetic force on the wire.

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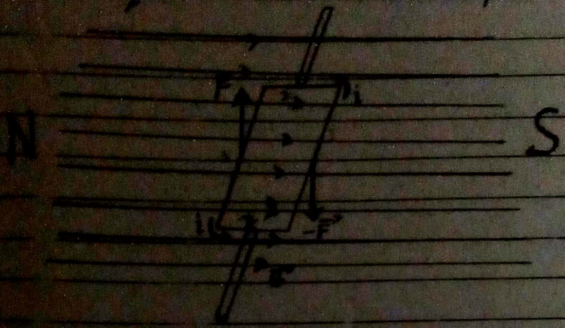
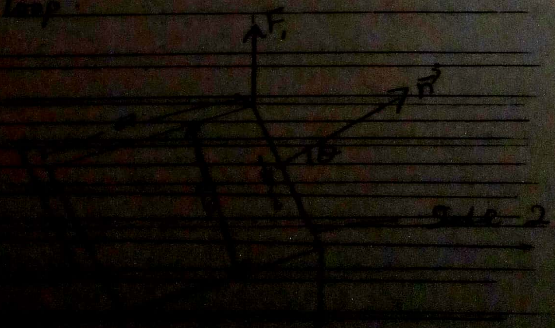
Question .... Write on both sides of the paper

Torque on a Current Loop



Consider a coil of wire carrying a current (a current-carrying) immersed in a magnetic field $\vec{B}$ as shown in the diagram above. The magnetic forces $\vec{F}$ and $-\vec{F}$ produce a torque on the coil tending to rotate it about its central axis. The coil is placed in the field so that long sides are $\perp$ar to the field direction but the short sides are not. To define the orientation of the loop in the magnetic field we use a normal vector $\vec{n}$ that is $\perp$ar to the plane of the loop.



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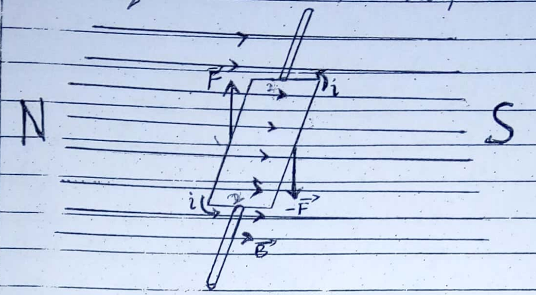
Question..........................   Write on both sides of the paper

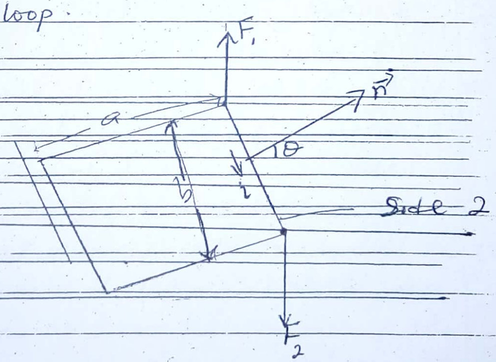
Torque on a Current Loop.



(a current-carrying loop)

Consider a coil of wire carrying a current, immersed in a magnetic field $\vec{B}$ as shown in the diagram above. Two magnetic forces $\vec{F}$ and $-\vec{F}$ produce a torque on the coil tending to rotate it about its central axis. The loop is placed in the field so that long sides are ⊥ar to the field direction but the short ~~st~~ sides are not.

To define the orientation of the loop in the magnetic field, we use a normal vector $\vec{n}$ that is ⊥ar to the plane of the loop.



The net force on the loop is the vector sum of the forces acting on its four sides. For side 2 and 4, the forces acting on them has the same magnitude but opposite

For sides 1 and 3, $\vec{L}$ is $\perp$ to $\vec{B}$ so the forces $\vec{F}_1$ and $\vec{F}_3$ have common magnitude iaB. The two forces (though cancel out) do not share the same line of action (they constitute a couple), so they do produce a net torque. The torque tends to rotate the loop so as to align its normal vector $\vec{n}$ with the direction of the magnetic field $\vec{B}$. The torque has moment arm $(b/2)\sin\theta$ about the central axis of the loop.

The magnitude of the torque τ' due to forces $\vec{F}_1$ and $\vec{F}_3$ is

$$\tau' = iaB\frac{b}{2}\sin\theta + iaB\frac{b}{2}\sin\theta$$

$$\tau' = iabB\sin\theta \quad \text{—— } 25$$

If the coil has N loops or turns and suppose that the turns are wound tightly enough that they can be approximated as all having the same dimensions and lying in a plane. The total torque on the coil then has magnitude

$$\tau = N\tau' = NiabB\sin\theta = (NiA)B\sin\theta \quad \text{—— } 26$$

$A = ab$ which is the area enclosed by the coil.

NiA are properties of the coil, Eqn 26 holds for all flat coil no matter the shape

Q.1 A single-turn current loop, carrying a current of 4.00 A, is in the shape of a right triangle with sides 50.0, 120.0, and 130.0 cm. The loop is in a uniform magnetic field of magnitude 75.0 mT whose direction is parallel to the current in the 130cm in the 130cm side of the loop (a) Find the magnitude of the magnetic force on each of the three sides of the loop. (b) Show that the total magnetic force on the loop is zero.

Q.2. A length L of wire carries a current i. Show

Question........................... Write on both sides of the paper

that if the wire is formed into a circular coil, then the maximum torque in a given magnetic field is developed when the coil has one turn only, and that maximum torque has the magnitude $\tau = L^2 i B/4\pi$.

From eqn 26, the torque τ is zero when angle $\theta = 0$ or $180°$. At this positn, the loop is the equilibrium. The effect of the torque τ is to rotate the loop in the directn of decreasing θ, i.e toward its equilibrium position.

Q.3 A circular coil of wire 8cm in diameter has 12 turns and carries a current of 5A. The coil is in a region where the magnetic field is 0.6T. (a) What is the maximum torque on the coil? (b) In what positn would the torque be one-half as great as in (a)

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Torque on a dipole is $\tau = pE\sin\theta = qE(l\sin\theta)$

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Question ...... $P = ql$

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In dis... 6Ho axts ... of forces

The Magnetic Dipole Moment

The current-carrying coil can be described with a single vector $\vec{\mu}$ called the magnetic dipole moment. The directn of $\vec{\mu}$ is taken to be the directn of normal vector $\vec{n}$ to the plane of the coil. Magnitude of $\vec{\mu}$ is defined as

$$\mu = NiA \qquad \text{--- (27)}$$

where N is the number of turns in the coil, i is the current through the coil, and A is the area enclosed by each turn of the coil. The unit of μ is Am^2.

Using equation (27), eqn (26) can be written as

$$\tau = \mu B\sin\theta \qquad \text{--- 28}$$

where θ is the angle between the vector $\vec{\mu}$ and $\vec{B}$.

In a vector relation, we write (28) as

$$\vec{\tau} = \vec{\mu}\times\vec{B} \qquad \text{--- (29)}$$

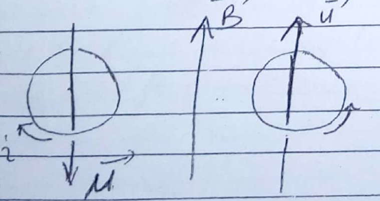
Eqn (29) is ~~the corresponding to eq~~ the eqn for torque exerted by an electric field dipole

$$\vec{\tau} = \vec{P}\times\vec{E}.$$

A magnetic dipole in an external magnetic field has a magnetic potential energy that depends on the dipole's orientatn in the field

$$U(\theta) = -\vec{\mu}\cdot\vec{B} \qquad \text{--- 30}$$

A magnetic dipole has its lowest energy ($U(\theta) = -\mu B\cos 0^\circ = -\mu B$) when its dipole moment $\vec{\mu}$ is lined up with the magnetic field. It has its highest energy ($U(\theta) = -\mu B\cos 180^\circ = +\mu B$) when $\vec{\mu}$ is directed opposite the field. This is shown below.



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Question........ .... Write on both sides of the paper

When a magnetic dipole rotates from an initial orientation θ_i to another orientation θ_f, the work W done on the dipole by the magnetic field is

$$W = -\Delta U = -(U_f - U_i) \quad \text{--- (31)}$$

where U_f and U_i are calculated from eqn (3).

Q1. The magnetic dipole moment of the Earth is 8.00×10^{22} J/T. Assume that this is produced by charges flowing in Earth's molten outer core. If the radius of their circular path is 3500km, cal. the current they produce.

Q2. A circular coil of 160 turns has a radius of 1.90cm. (a) cal. the current that results in a magnetic dipole moment of $2.30\ A\cdot m^2$ (b) find the maximum torque that the coil, carrying this current, can experience in a uniform 35.0mT magnetic field.

Q3. A circular loop of wire having a radius of 8.0 cm carries a current of 0.20A. A vector of unit length and parallel to the dipole moment $\vec{\mu}$ of loop is given by $0.60\hat{i} - 0.80\hat{j}$. If the loop is located in a uniform magnetic field given by $\vec{B} = (0.25T)\hat{i} + (0.30T)\hat{k}$, find (a) the torque on the loop (in unit-vector notation) and (b) the magnetic potential energy on the loop.

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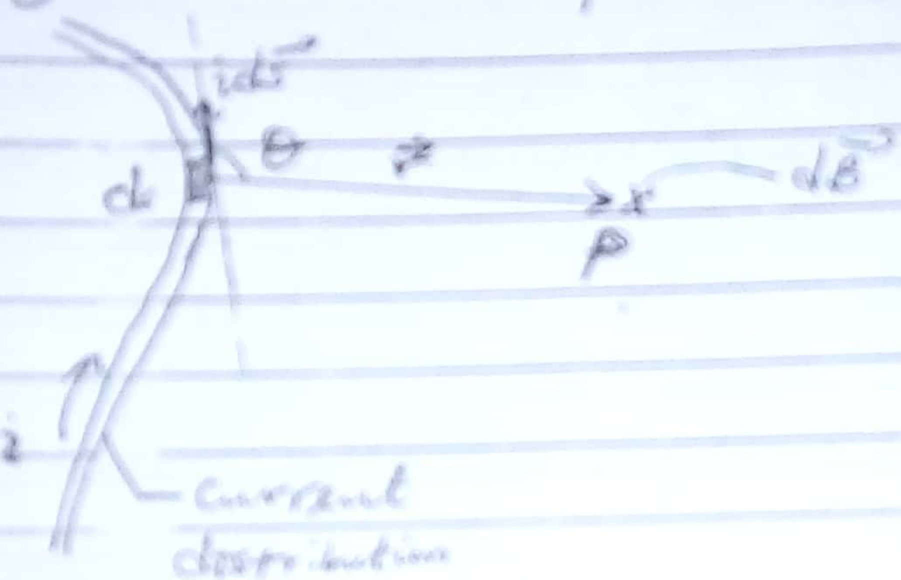
produced by a given distribution of charged ptcs



The figure above shows a wire of arbitrary shape carry a current i. Our interest is to find the magnetic field $\vec{B}$ at a nearby point P. We divide the wire into different elements ds and then define for each element a length vector $\vec{ds}$ that has length ds and whose direction is the direction current in ds. Then we can define the a differential current element to be $i\vec{ds}$. We wish to calculate the field $d\vec{B}$ produced at P by a typical current-length element. Like electric field, experiment shows that magnetic fields can be superimposed to find a net field. Thus, we can cal the net field $\vec{B}$ at P by summing, via integration, the contributions $d\vec{B}$ from all the current-length elements. The magnitude of the field $d\vec{B}$ at point P by a current length element $i\vec{ds}$ is

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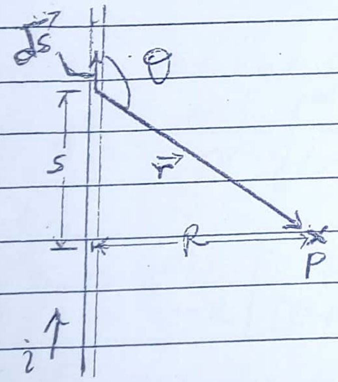
Where θ is the angle between the directions of $\vec{ds}$ and $\vec{r}$, the vector that extends from ds to P. μ_0 is a constant called the permeability constant.

The direction of $\vec{dB}$ is that of the cross product $\vec{ds} \times \vec{r}$. We can therefore write eqn (32) in vector form as

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, \vec{ds} \times \vec{r}}{r^3} \quad \text{--- (33)}$$

Eqn (32) and (33) are known as the law of Biot and Savart which is experimentally deduced is an inverse-square law. We shall use this law to cal. the net magnetic field $\vec{B}$ produced at a point by various distributions of current.

Magnetic Field Due to a Current in a Long Straight Wire.



directn is $\vec{B}$ ... from ... hand rule

The figure above is just like the previous one except that now the wire is straight and of infinite length. We want the field $\vec{B}$ at point P, a lar distance R from the wire. The magnitude of the differential magnetic field produced at P by the current-length element $i\vec{ds}$ located at distance r from P is given by eqn (32)

$$dB = \frac{\mu_0}{4\pi} \frac{i\, ds \sin\theta}{r^2}$$

The directn of $\vec{dB}$ in the figure above is that of the vector $\vec{ds} \times \vec{r}$.

Note that $\vec{dB}$ at point P has this same directn for all the current-length elements into which the wire can be divided. Thus, we can find the magnitude of the magnetic field produced at P by the current-length elements in the upper half of the infinitely long wire by integrating dB from 0 to ∞.

If we consider a current-length element in the lower half of the wire, one that is as far below P as $d\vec{s}$ is above P. from Eqn (27) we have total magnetic field $\vec{B}$ at P to be

$$B = 2\int_0^{\infty} dB = \frac{\mu_0 i}{2\pi}\int_0^{\infty}\frac{\sin\theta\, ds}{r^2} \quad (28)$$

but variables θ, s and r are related by

$$r = \sqrt{s^2 + R^2}$$

$$\sin\theta = \sin(\pi - \theta) = \frac{R}{\sqrt{s^2+R^2}}$$

$d(\tan\theta) = \sec^2\theta$

Put these in eqn 28, we have

$$B = \frac{\mu_0 i}{2\pi}\int_0^{\infty}\frac{R\, ds}{(s^2+R^2)^{3/2}}$$

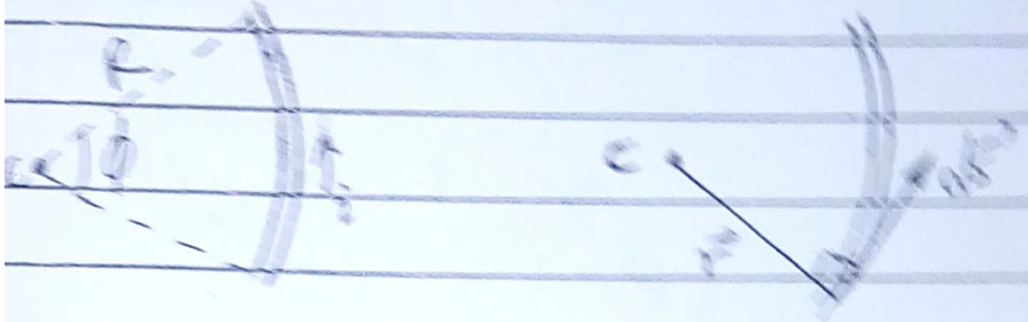
$$B = \frac{\mu_0 i}{2\pi R}\left[\frac{s}{(s^2+R^2)^{1/2}}\right]_0^{\infty} = \frac{\mu_0 i}{2\pi R} \quad (29)$$

Note that the magnetic field at P due to either the lower half or the upper half of the infinite wire is half that value

$$B = \frac{\mu_0 i}{4\pi R} \quad \text{semi-infinite straight wire} \quad (30)$$

Question .... Write on both sides of the paper

Magnetic Field Due to a Current in a Circular Arc of Wire



To find the magnetic field produced at a point by a current in a curved wire, we use eqn (29) to write the magnitude of the field produced by a single current-length element and again integrate to find the net field produced by all the current-length elements.

The figure above shows an arc-shaped wire with central angle ϕ, radius R and centre C carrying current i. Applying eqn (29), the angle θ b/w the vectors $d\vec{s}$ and $\hat{r}$ is $90°$; $r = R$

So eqn (29) becomes we have

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\,\sin 90°}{R^2} = \frac{\mu_0}{4\pi}\frac{i\,ds}{R^2}$$

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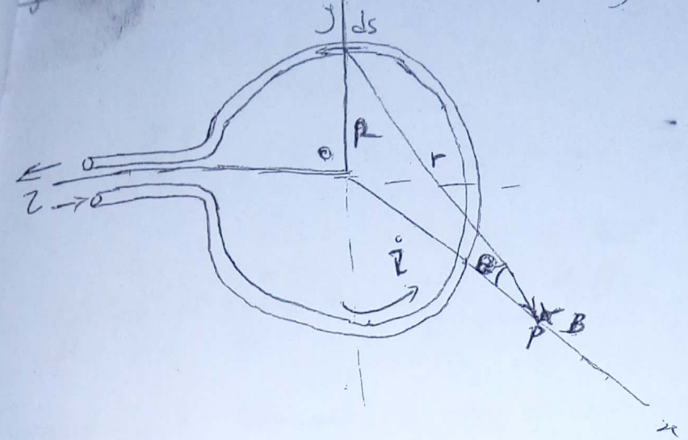
Magnetic Field of a Circular Loop

5



The figure above shows a circular loop of wire of radius R, carrying current i that is led into and out of the loop through long straight wires side by side. The currents in the straight wires are in opposite directions and annul each other's magnetic effects. We can use eqn (32) to calculate the magnetic field at a point P along a line perpendicular to the plane of the loop, a distance x from ~~the~~ it's center.

from the diagram, $r = (x^2 + R^2)^{1/2}$.

$$dB = \frac{\mu_0}{4\pi} \frac{i\,ds \sin\theta}{x^2 + R^2}$$

but $\sin\theta = \frac{R}{r} = \frac{R}{x^2 + R^2}$

$$dB = \frac{\mu_0}{4\pi} \frac{i\,ds\,R}{(x^2 + R^2)^{3/2}}$$

but $ds = R\,d\phi$

$$dB = \frac{\mu_0}{4\pi} \frac{i R^2\,d\phi}{(x^2 + R^2)^{3/2}}$$

$$B = \int dB = \frac{\mu_0}{4\pi} \frac{i R^2}{(x^2 + R^2)^{3/2}} \int d\phi = \frac{\mu_0}{4\pi} \frac{i R^2}{(x^2 + R^2)^{3/2}} \phi \quad \text{but} \quad (39)$$

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$\phi = 2\pi$, eqn (34) becomes . . 6

$$B = \frac{\mu_0 i R^2}{2(x^2+R^2)^{3/2}} \quad - - - - \quad 40$$

At the centre of the loop, $x = 0$, and eqn 40 reduces to

$$B = \frac{\mu_0 i}{2R} \quad \text{~~which is the~~} \quad (41)$$

If the coil has N turns the $B = \frac{\mu_0 N i}{2R} \quad - - - \quad (42)$

eqn 41 and 38 are the same

1. Considering the magnetic field along the axis of a circular loop of radius R, at what distance from the center of the loop is the field $\frac{1}{10}$ of its value at the center?

2. A circular coil of radius 5cm has 200 turns and carries a current of 0.2A. What is the magnetic field

(a) at the center of the coil?

(b) At a point on the axis of the coil, 10cm from its center?

A closely wound coil has a diameter of 40cm and carries a current of 2.5A. How many turns does it have if the magnetic field at the center of the coil is 1.26×10^{-4} T?

Question .... Write on both sides of the paper

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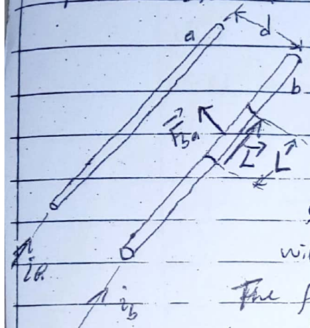
ϕ becomes 2π, we have

$$B = \frac{\mu_0 i (2\pi)}{4\pi R} - \frac{\mu_0 i}{2R} \quad --- (38)$$

Force Between Two Parallel Current-Carrying Wires



A current-carrying wire has a magnetic field round it. If two long parallel wires carrying currents ~~exert~~ are placed close to each other as shown in the diagram above, they will ~~exert~~ exert a force on each other.

The figure above shows two wires, separated by a distance d and carrying current i_a and i_b.

Let look at the force on wire b due to the current in wire a. Current i_a produces a magnetic field $\vec{B_a}$, The field B_a causes a force on b. To find the force on b, we need the magnitude and direction of the field $\vec{B_a}$ at the side of wire b.

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

Right-hand rule tells us the direction of $\vec{B}$

To find the force on wire b carrying current i_b, we use eqn 23 $F = i\vec{L} \times \vec{B}$

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B_a} \quad --- (43)$$

Where $\vec{L}$ is the length vector of the wire. From the diagram above, $\vec{L}$ and $\vec{B_a}$ are $\perp$ar

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d} \quad (44)$$

The direction of $\vec{F}_{ba}$ is the direction of cross product

$\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$ 8

Question .... Write on both sides of the paper

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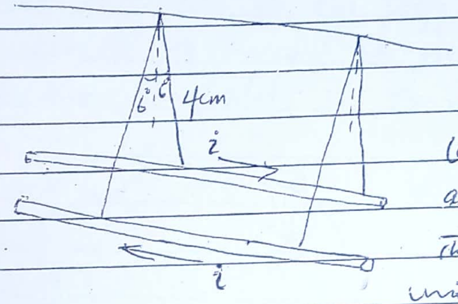
$\vec{L} \times \vec{B_a}$. Applying the left hand rule, $\vec{F_{ba}}$ is directly toward wire a, as shown.

The same procedure can be used to compute the force on wire a due to the current in wire b. We would find that the force is directly toward wire b; hence, the two wires with parallel currents attract each other.

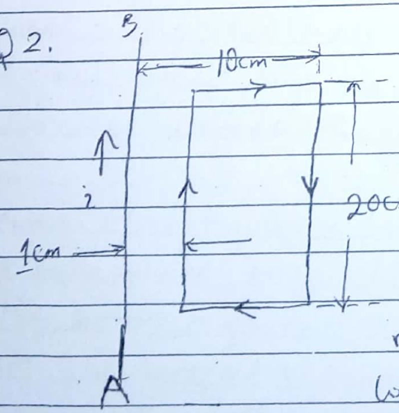
i.e Parallel currents attract, and antiparallel currents repel.

Q1



Two long parallel wires are hung by cords of 4cm length from a common axis, as shown in the diagram. The wires have a mass per unit length of 50g.m⁻¹ and carry the same current in opposite directions. What is the current if the cords hang at an angle 6° with the vertical?

Q2.



The long straight wire AB in the diagram carries a current of 20A. The rectangular loop whose long edges are parallel to the wire carries a current of 10A. Find the magnitude and direction of the resultant force exerted on the loop by the magnetic field of the wire.

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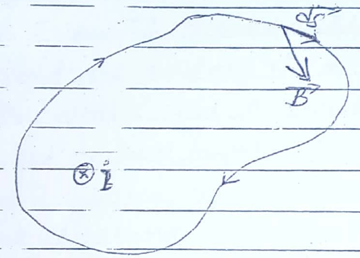
Ampere's Law

Similar to Gauss' law, Ampere's law can be used to find the net magnetic field due to any distribution of currents if only the distribution has some ~~sym~~ symmetry.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

The circle on the integral sign indicates that the integration is over a closed loop called an Amperian loop. i_{enc} is the net current encircled by the loop. If a closed loop does not encircle a current-carrying wire, the line integral of the B field is zero.



Ampere's law: The line integral of the magnetic field B around any closed loop is equal to μ_0 times the net current across the area bounded by the loop.

Applications of Ampere's law

1. If B is everywhere tangent to the integration path or loop and has the same magnitude B at every point on the path, then its line integral is equal to B multiplied by the circumference of the path.

If B is everywhere far to the path, for all or some portion of the path or loop, that portion of the path makes

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Question ....

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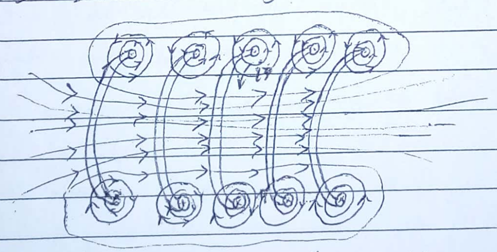
no contribution to the line integral.

3. In the integral $\oint B \cdot dl$, B is always the total magnetic field at each point on the path. In general this field is caused partly by currents linked by the path and partly by currents outside. Even when no current is linked by the path, the field at points on the path need not be zero. In that case, however, $\oint B \cdot dl$ is always zero.

4. Some judgment is required in choosing an integration path. Two useful guiding principles are that the point or points at which the field is to be determined must lie on the path, and that the path must have enough symmetry so that the integral can be evaluated.

Field of a Solenoid.

A Solenoid is constructed by winding wire in a helix around the surface of a cylindrical form. The turns of the winding are ordinarily closely spaced and may consist of one or more layers. Consider a Solenoid or relatively small number of circular turns each carrying current I as shown in diagram below.



A vertical cross section through the central axis of a stretched-out Solenoid.

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Solenoid carrying a current i.

Applicatn of Ampere's law.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

to an ideal solenoid shown above, where $\vec{B}$ is uniform within the solenoid and zero outside it, using the rectangular Amperian loop abcda. We write $\oint \vec{B} \cdot d\vec{s}$ as the sum of four integrals, one for each loop segment:

$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_d^a \vec{B} \cdot d\vec{s}$$

The first integral on the right of eqn -- is Bh, where B is the magnitude of the uniform field $\vec{B}$ inside the solenoid and h is the arbitrary length of the segment from a to b. The second and fourth integrals are zero because for every element ds of these segments, $\vec{B}$ ~~eith~~ is either lar to ds or zero, and thus the product $\vec{B} \cdot d\vec{s}$ is zero. The third integral, which is taken along a segment that lie outside the solenoid, is zero because $B = 0$ at all external points. Thus, $\oint \vec{B} \cdot d\vec{s}$ for the entire rectangular loop has the value Bh.

The net current encircled by the rectangular loop i_{enc} is $i(nh)$

$$i_{enc} = i(nh)$$

where n is the number of turns per unit length of the solenoid and ~~the number of loop encloses nh turns~~ nh is the number of turn encloses by the loop

$$Bh = \mu_0 i n h$$

$$B = \mu_0 i n \quad \text{for idea solenoid} \; --$$